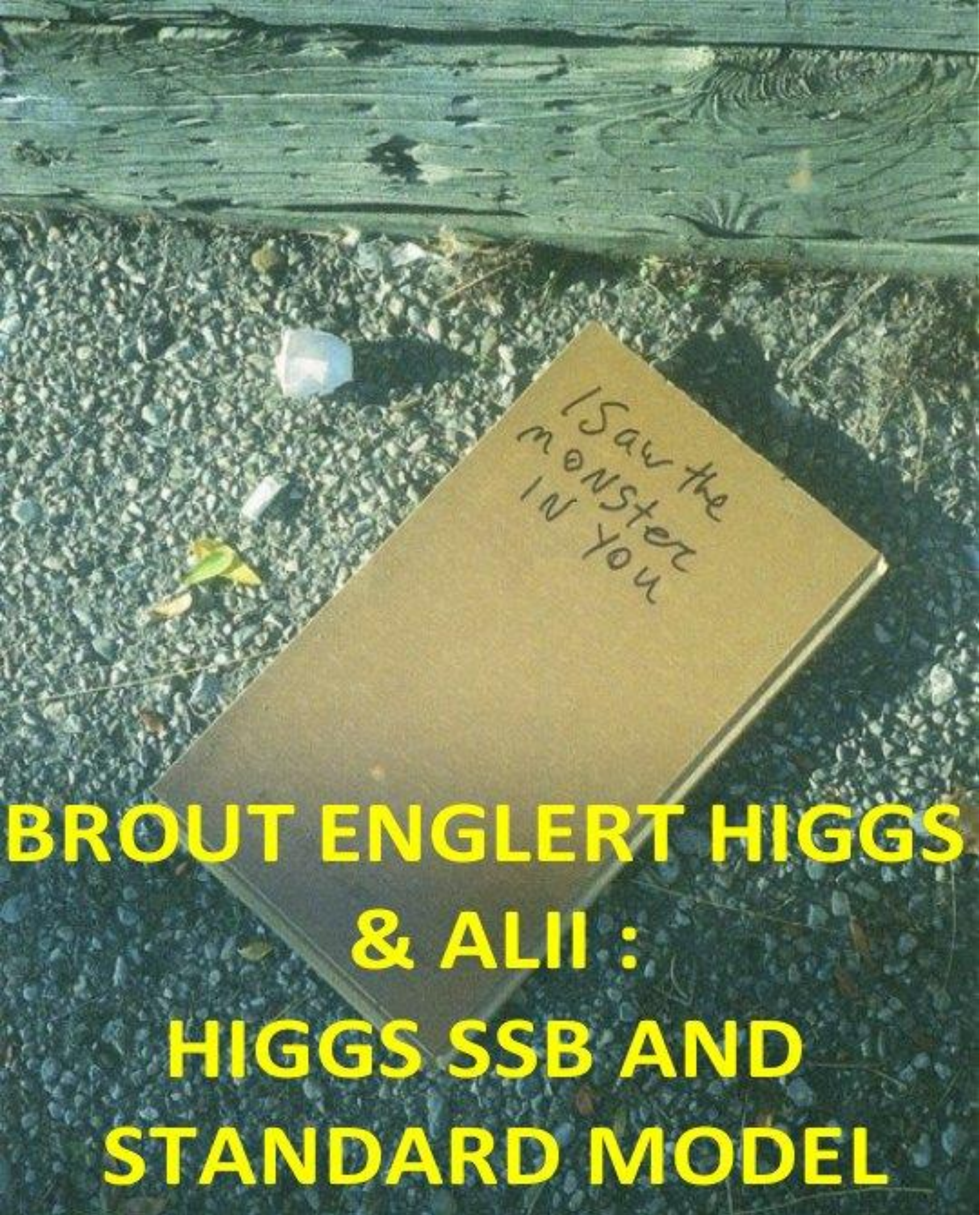

BROUT ENGLERT HIGGS & ALII : HIGGS SSB AND STANDARD MODEL

*Work supported in part by the U. S. Atomic Energy Commission and in part by the Graduate School from funds supplied by the Wisconsin Alumni Research Foundation.

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BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

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It is of interest to inquire whether gauge vector mesons acquire mass through interaction[1]; by a gauge vector meson we mean a Yang-Mills field[2] associated with the extension of a Lie group from global to local symmetry. The importance of this problem resides in the possibility that strong-interaction physics originates from massive gauge fields related to a system of conserved currents.[3] In this note, we shall show that in certain cases vector mesons do indeed acquire mass when the vacuum is degenerate with respect to a compact Lie group.

Theories with degenerate vacuum (broken symmetry) have been the subject of intensive study since their inception by Nambu.[4-6] A characteristic feature of such theories is the possible existence of zero-mass bosons which tend to restore the symmetry.[7,8] We shall show that it is precisely these singularities which maintain the gauge invariance of the theory, despite the fact that the vector meson acquires mass.

We shall first treat the case where the original fields are a set of bosons φ_A which transform as a basis for a representation of a compact Lie group. This example should be considered as a rather general phenomenological model. As such, we shall not study the particular mechanism by which the symmetry is broken but simply assume that such a mechanism exists. A calculation performed in lowest order perturbation theory indicates that those vector mesons which are coupled to currents that "rotate" the original vacuum are the ones which acquire mass [see Eq. (6)].

We shall then examine a particular model based on chirality invariance which may have a more fundamental significance. Here we begin with a chirality-invariant Lagrangian and introduce both vector and pseudovector gauge fields, thereby guaranteeing invariance under both local phase and local γ_5-phase transformations. In this model the gauge fields themselves may break the γ_5 invariance leading to a mass for the original Fermi field. We shall show in this case that the pseudovector field acquires mass.

In the last paragraph we sketch a simple argument which renders these results reasonable.

(1) Lest the simplicity of the argument be shrouded in a cloud of indices, we first consider a one-parameter Abelian group, representing, for example, the phase transformation of a charged boson; we then present the generalization to an arbitrary compact Lie group.

The interaction between the φ and the A_μ fields is

$$H_{\rm int} = ieA_\mu \varphi^* \overleftrightarrow{\partial}_\mu \varphi - e^2 \varphi^* \varphi A_\mu A_\mu, \qquad (1)$$

where $\varphi = (\varphi_1 + i\varphi_2)/\sqrt{2}$. We shall break the symmetry by fixing $\langle\varphi\rangle \neq 0$ in the vacuum, with the phase chosen for convenience such that $\langle\varphi\rangle = \langle\varphi^*\rangle = \langle\varphi_1\rangle/\sqrt{2}$.

We shall assume that the application of the

theorem of Goldstone, Salam, and Weinberg[7] is straightforward and thus that the propagator of the field φ_2, which is "orthogonal" to φ_1, has a pole at $q=0$ which is not isolated.

We calculate the vacuum polarization loop $\Pi_{\mu\nu}$ for the field A_μ in lowest order perturbation theory about the self-consistent vacuum. We take into consideration only the broken-symmetry diagrams (Fig. 1). The conventional terms do not lead to a mass in this approximation if gauge invariance is carefully maintained. One evaluates directly

$$\Pi_{\mu\nu}(q)=(2\pi)^4 ie^2[g_{\mu\nu}\langle\varphi_1\rangle^2-(q_\mu q_\nu/q^2)\langle\varphi_1\rangle^2]. \tag{2}$$

Here we have used for the propagator of φ_2 the value $[i/(2\pi)^4]/q^2$; the fact that the renormalization constant is 1 is consistent with our approximation.[9] We then note that Eq. (2) both maintains gauge invariance ($\Pi_{\mu\nu}q_\nu=0$) and causes the A_μ field to acquire a mass

$$\mu^2=e^2\langle\varphi_1\rangle^2. \tag{3}$$

We have not yet constructed a proof in arbitrary order; however, the similar appearance of higher order graphs leads one to surmise the general truth of the theorem.

Consider now, in general, a set of boson-field operators φ_A (which we may always choose to be Hermitian) and the associated Yang-Mills field $A_{a,\mu}$. The Lagrangian is invariant under the transformation[10]

$$\delta\varphi_A=\textstyle\sum_{a,A}\epsilon_a(x)T_{a,AB}\varphi_B,$$

$$\delta A_{a,\mu}=\textstyle\sum_{c,b}\epsilon_c(x)c_{acb}A_{b,\mu}+\partial_\mu\epsilon_a(x), \tag{4}$$

where c_{abc} are the structure constants of a compact Lie group and $T_{a,AB}$ the antisymmetric generators of the group in the representation defined by the φ_B.

Suppose that in the vacuum $\langle\varphi_{B'}\rangle\neq 0$ for some B'. Then the propagator of $\sum_{A,B'}T_{a,AB'}\varphi_A$

FIG. 1. Broken-symmetry diagram leading to a mass for the gauge field. Short-dashed line, $\langle\varphi_1\rangle$; long-dashed line, φ_2 propagator; wavy line, A_μ propagator. (a)$\to(2\pi)^4ie^2g_{\mu\nu}\langle\varphi_1\rangle^2$, (b)$\to-(2\pi)^4ie^2(q_\mu q_\nu/q^2)\times\langle\varphi_1\rangle^2$.

$\times\langle\varphi_{B'}\rangle$ is, in the lowest order,

$$\left[\frac{i}{(2\pi)^4}\right]\sum_{A,B',C'}\frac{T_{a,AB'}\langle\varphi_{B'}\rangle T_{a,AC'}\langle\varphi_{C'}\rangle}{q^2}$$

$$\equiv\left[\frac{-i}{(2\pi)^4}\right]\frac{(\langle\varphi\rangle T_aT_a\langle\varphi\rangle)}{q^2}.$$

With λ the coupling constant of the Yang-Mills field, the same calculation as before yields

$$\Pi_{\mu\nu}{}^a(q)=-i(2\pi)^4\lambda^2(\langle\varphi\rangle T_aT_a\langle\varphi\rangle)$$

$$\times[g_{\mu\nu}-q_\mu q_\nu/q^2],$$

giving a value for the mass

$$\mu_a{}^2=-(\langle\varphi\rangle T_aT_a\langle\varphi\rangle). \tag{6}$$

(2) Consider the interaction Hamiltonian

$$H_{\text{int}}=-\eta\bar\psi\gamma_\mu\gamma_5\psi B_\mu-\epsilon\bar\psi\gamma_\mu\psi A_\mu, \tag{7}$$

where A_μ and B_μ are vector and pseudovector gauge fields. The vector field causes attraction whereas the pseudovector leads to repulsion between particle and antiparticle. For a suitable choice of ϵ and η there exists, as in Johnson's model,[11] a broken-symmetry solution corresponding to an arbitrary mass m for the ψ field fixing the scale of the problem. Thus the fermion propagator $S(p)$ is

$$S^{-1}(p)=\gamma p-\Sigma(p)=\gamma p[1-\Sigma_2(p^2)]-\Sigma_1(p^2), \tag{8}$$

with

$$\Sigma_1(p^2)\neq 0$$

and

$$m[1-\Sigma_2(m^2)]-\Sigma_1(m^2)=0.$$

We define the gauage-invariant current $J_\mu{}^5$ by using Johnson's method[12]:

$$J_\mu{}^5=-\eta\lim_{\xi\to 0}\bar\psi'(x+\xi)\gamma_\mu\gamma_5\psi'(x),$$

$$\psi'(x)=\exp[-i\textstyle\int_{-\infty}^x\eta B_\mu(y)dy^\mu\gamma_5]\psi(x). \tag{9}$$

This gives for the polarization tensor of the

pseudovector field

$$\Pi_{\mu\nu}{}^5(q) = \eta^2 \frac{i}{(2\pi)^4} \int \mathrm{Tr}\{S(p-\tfrac{1}{2}q)\Gamma_{\nu 5}(p-\tfrac{1}{2}q; p+\tfrac{1}{2}q) \times S(p+\tfrac{1}{2}q)\gamma_\mu\gamma_5 - S(p)[\partial S^{-1}(p)/\partial p_\nu]S(p)\gamma_\mu\} d^4p, \quad (10)$$

where the vertex function $\Gamma_{\nu 5} = \gamma_\nu\gamma_5 + \Lambda_{\nu 5}$ satisfies the Ward identity[5]

$$q_\nu \Lambda_{\nu 5}(p-\tfrac{1}{2}q; p+\tfrac{1}{2}q) = \Sigma(p-\tfrac{1}{2}q)\gamma_5 + \gamma_5\Sigma(p+\tfrac{1}{2}q), \quad (11)$$

which for low q reads

$$q_\nu\Gamma_{\nu 5} = q_\nu\gamma_\nu\gamma_5[1-\Sigma_2] + 2\Sigma_1\gamma_5 - 2(q_\nu p_\nu)(\gamma_\lambda p_\lambda)(\partial\Sigma_2/\partial p^2)\gamma_5. \quad (12)$$

The singularity in the longitudinal $\Gamma_{\nu 5}$ vertex due to the broken-symmetry term $2\Sigma_1\gamma_5$ in the Ward identity leads to a nonvanishing gauge-invariant $\Pi_{\mu\nu}{}^5(q)$ in the limit $q\to 0$, while the usual spurious "photon mass" drops because of the second term in (10). The mass of the pseudovector field is roughly $\eta^2 m^2$ as can be checked by inserting into (10) the lowest approximation for $\Gamma_{\nu 5}$ consistant with the Ward identity.

Thus, in this case the general feature of the phenomenological boson system survives. We would like to emphasize that here the symmetry is broken through the gauge fields themselves. One might hope that such a feature is quite general and is possibly instrumental in the realization of Sakurai's program.[3]

(3) We present below a simple argument which indicates why the gauge vector field need not have zero mass in the presence of broken symmetry. Let us recall that these fields were introduced in the first place in order to extend the symmetry group to transformations which were different at various space-time points. Thus one expects that when the group transformations become homogeneous in space-time, that is $q\to 0$, no dynamical manifestation of these fields should appear. This means that it should cost no energy to create a Yang-Mills quantum at $q=0$ and thus the mass is zero. However, if we break gauge invariance of the first kind and still maintain gauge invariance of the second kind this reasoning is obviously incorrect. Indeed, in Fig. 1, one sees that the A_μ propagator connects to intermediate states, which are "rotated" vacua. This is seen most clearly by writing $\langle\varphi_1\rangle = \langle[Q\varphi_2]\rangle$ where Q is the group generator. This effect cannot vanish in the limit $q\to 0$.

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BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

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In a recent note[1] it was shown that the Goldstone theorem,[2] that Lorentz-covariant field theories in which spontaneous breakdown of symmetry under an internal Lie group occurs contain zero-mass particles, fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The purpose of the present note is to report that, as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the coupling tends to zero. This phenomenon is just the relativistic analog of the plasmon phenomenon to which Anderson[3] has drawn attention: that the scalar zero-mass excitations of a superconducting neutral Fermi gas become longitudinal plasmon modes of finite mass when the gas is charged.

The simplest theory which exhibits this behavior is a gauge-invariant version of a model used by Goldstone[2] himself: Two real[4] scalar fields φ_1, φ_2 and a real vector field A_μ interact through the Lagrangian density

$$L=-\tfrac{1}{2}(\nabla\varphi_1)^2-\tfrac{1}{2}(\nabla\varphi_2)^2 -V(\varphi_1^{\,2}+\varphi_2^{\,2})-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{1}$$

where

$$\nabla_\mu\varphi_1=\partial_\mu\varphi_1-eA_\mu\varphi_2,$$

$$\nabla_\mu\varphi_2=\partial_\mu\varphi_2+eA_\mu\varphi_1,$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu,$$

e is a dimensionless coupling constant, and the metric is taken as $-+++$. L is invariant under simultaneous gauge transformations of the first kind on $\varphi_1\pm i\varphi_2$ and of the second kind on A_μ. Let us suppose that $V'(\varphi_0^{\,2})=0$, $V''(\varphi_0^{\,2})>0$; then spontaneous breakdown of U(1) symmetry occurs. Consider the equations [derived from (1) by treating $\Delta\varphi_1$, $\Delta\varphi_2$, and A_μ as small quantities] governing the propagation of small oscillations about the "vacuum" solution $\varphi_1(x)=0$, $\varphi_2(x)=\varphi_0$:

$$\partial^\mu\{\partial_\mu(\Delta\varphi_1)-e\varphi_0A_\mu\}=0, \tag{2a}$$

$$\{\partial^2-4\varphi_0^{\,2}V''(\varphi_0^{\,2})\}(\Delta\varphi_2)=0, \tag{2b}$$

$$\partial_\nu F^{\mu\nu}=e\varphi_0\{\partial^\mu(\Delta\varphi_1)-e\varphi_0A_\mu\}. \tag{2c}$$

Equation (2b) describes waves whose quanta have (bare) mass $2\varphi_0\{V''(\varphi_0^{\,2})\}^{1/2}$; Eqs. (2a) and (2c) may be transformed, by the introduction of new variables

$$B_\mu=A_\mu-(e\varphi_0)^{-1}\partial_\mu(\Delta\varphi_1),$$
$$G_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu=F_{\mu\nu}, \tag{3}$$

into the form

$$\partial_\mu B^\mu=0,\quad \partial_\nu G^{\mu\nu}+e^2\varphi_0^{\,2}B^\mu=0. \tag{4}$$

Equation (4) describes vector waves whose quanta have (bare) mass $e\varphi_0$. In the absence of the gauge field coupling ($e=0$) the situation is quite different: Equations (2a) and (2c) describe zero-mass scalar and vector bosons, respectively. In passing, we note that the right-hand side of (2c) is just the linear approximation to the conserved current: It is linear in the vector potential, gauge invariance being maintained by the presence of the gradient term.[5]

When one considers theoretical models in which spontaneous breakdown of symmetry under a semisimple group occurs, one encounters a variety of possible situations corresponding to the various distinct irreducible representations to which the scalar fields may belong; the gauge field always belongs to the adjoint representation.[6] The model of the most immediate interest is that in which the scalar fields form an octet under SU(3): Here one finds the possibility of two nonvanishing vacuum expectation values, which may be chosen to be the two $Y=0$, $I_3=0$ members of the octet.[7] There are two massive scalar bosons with just these quantum numbers; the remaining six components of the scalar octet combine with the corresponding components of the gauge-field octet to describe

massive vector bosons. There are two $I=\frac{1}{2}$ vector doublets, degenerate in mass between $Y=\pm1$ but with an electromagnetic mass splitting between $I_3=\pm\frac{1}{2}$, and the $I_3=\pm1$ components of a $Y=0$, $I=1$ triplet whose mass is entirely electromagnetic. The two $Y=0$, $I=0$ gauge fields remain massless: This is associated with the residual unbroken symmetry under the Abelian group generated by Y and I_3. It may be expected that when a further mechanism (presumably related to the weak interactions) is introduced in order to break Y conservation, one of these gauge fields will acquire mass, leaving the photon as the only massless vector particle. A detailed discussion of these questions will be presented elsewhere.

It is worth noting that an essential feature of the type of theory which has been described in this note is the prediction of incomplete multiplets of scalar and vector bosons.[8] It is to be expected that this feature will appear also in theories in which the symmetry-breaking scalar fields are not elementary dynamic variables but bilinear combinations of Fermi fields.[9]

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[4]In the present note the model is discussed mainly in classical terms; nothing is proved about the quantized theory. It should be understood, therefore, that the conclusions which are presented concerning the masses of particles are conjectures based on the quantization of linearized classical field equations. However, essentially the same conclusions have been reached independently by F. Englert and R. Brout, Phys. Rev. Letters 13, 321 (1964): These authors discuss the same model quantum mechanically in lowest order perturbation theory about the self-consistent vacuum.

[5]In the theory of superconductivity such a term arises from collective excitations of the Fermi gas.

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[7]These are just the parameters which, if the scalar octet interacts with baryons and mesons, lead to the Gell-Mann–Okubo and electromagnetic mass splittings: See S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964).

[8]Tentative proposals that incomplete SU(3) octets of scalar particles exist have been made by a number of people. Such a rôle, as an isolated $Y=\pm1$, $I=\frac{1}{2}$ state, was proposed for the κ meson (725 MeV) by Y. Nambu and J. J. Sakurai, Phys. Rev. Letters 11, 42 (1963). More recently the possibility that the σ meson (385 MeV) may be the $Y=I=0$ member of an incomplete octet has been considered by L. M. Brown, Phys. Rev. Letters 13, 42 (1964).

[9]In the theory of superconductivity the scalar fields are associated with fermion pairs; the doubly charged excitation responsible for the quantization of magnetic flux is then the surviving member of a U(1) doublet.

SPLITTING OF THE 70-PLET OF SU(6)

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1. In a previous note,[1] hereafter called I, we proposed an expression for the mass operator responsible for lifting the degeneracies of spin-unitary spin supermultiplets [Eq. (31)-I]. The purpose of the present note is to apply this expression to the 70-dimensional representation of SU(6).

The importance of the 70-dimensional representation has already been underlined by Pais.[2] Since

$$\underline{35}\otimes\underline{56}=\underline{56}\oplus\underline{70}\oplus\underline{700}\oplus\underline{1134}, \tag{1}$$

it follows that $\underline{70}$ is the natural candidate for accommodating the higher meson-baryon resonances. Furthermore, since the SU(3)⊗SU(2) content is

$$\underline{70}=(\underline{1},\underline{2})+(\underline{8},\underline{2})+(\underline{10},\underline{2})+(\underline{8},\underline{4}), \tag{2}$$

we may assume that partial occupancy of the $\underline{70}$ representation has already been established through the so-called γ octet[2] $(\frac{3}{2})^-$. Recent experiments appear to indicate that some $(\frac{1}{2})^-$ states may also be at hand.[3] With six masses at one's disposal, our formulas can predict the masses of all the other occupants of $\underline{70}$ and also provide a consistency check on the input. Our discussion of the $\underline{70}$ representation thus appears to be of immediate physical interest.

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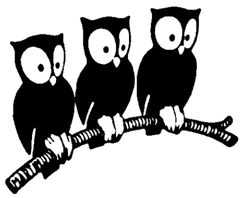
Aspects non-perturbatifs des théories de jauge (supersymétriques)



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Résumé

À un niveau élémentaire, on présente certains aspects non-perturbatifs des théories de jauge non-abéliennes à quatre dimensions d'espace-temps. Des résultats rigoureux ont pu être obtenus dans le cadre des théories supersymétriques, mettant en évidence une physique très riche liée à la dynamique des champs de jauge en couplage fort. Les nouveaux phénomènes qui sont apparus joueront peut-être un rôle dans l'élaboration des modèles phénoménologiques du futur. Néanmoins, l'application quantitative de ces idées à des théories non-supersymétrique, telle que la chromodynamique quantique, reste hors de portée pour le moment.

Les théories que nous considérons partagent de nombreuses caractéristiques avec les théories réalistes à la base du modèle standard. Principalement, elles contiennent des champs de jauge non-abéliens, éventuellement couplés à la matière, et sont asymptotiquement libres. De plus, il existe des corrections non-perturbatives à la série de perturbation. Semi-classiquement, ces corrections sont dues aux instantons, qui sont présents dans toutes les théories de jauge non-abéliennes. La symétrie de jauge est spontanément brisée par la valeur moyenne non-nulle d'un champ scalaire complexe ϕ, qui appartient à la représentation adjointe du groupe de jauge, et qui joue le rôle du boson de Higgs. Génériquement, le groupe de jauge G est brisé en $U(1)^r$ où r est le rang du groupe. Je voudrais souligner de plus quatre caractéristiques particulières des théories supersymétriques $N=2$, qui sont celles pour lesquelles des résultats quantitatifs rigoureux peuvent être obtenus pour le spectre de masse et le contenu en particules, et qu'il est bon de garder à l'esprit. Tout d'abord, il n'existe pas de fermions chiraux. Ensuite, la série de perturbation s'arrête à une boucle; il n'y a donc aucun problème de resommation, et l'on peut considérer sans difficulté d'éventuelles corrections non-perturbatives. Troisièmement, ces corrections non-perturbatives peuvent être calculées semi-classiquement sans ambiguité; les problèmes infrarouges venant de l'intégration sur la taille de l'instanton, bien connus en QCD, sont parfaitement maîtrisés ici. Enfin, le potentiel scalaire est du type $\mathrm{tr}\,[\phi,\phi^\dagger]^2$ et a donc des directions plates, qui ne sont pas levées au niveau quantique. Ceci veut dire que la valeur moyenne du Higgs n'est pas déterminée mais doit être considérée comme un paramètre supplémentaire de la théorie. Sa donnée fixe une échelle d'énergie, à comparer à l'échelle Λ générée dynamiquement en raison de la liberté asymptotique. Le régime de couplage fort est celui pour lequel $|\langle\phi\rangle|$ est de l'ordre, ou plus petit, que Λ.

Une question de physique évidente à poser une fois le lagrangien d'une théorie connu est la suivante: quelles sont les particules et leurs caractéristiques (masse, spin, etc...) qu'un expérimentateur pourra éventuellement détecter dans un monde décrit par la théorie étudiée? On ne peut répondre en général à cette question en lisant simplement le lagrangien. Même à un niveau perturbatif, de gros efforts ont été nécessaires dans le passé pour comprendre comment une théorie de jauge non-abélienne pouvait contenir des particules *massives* de spin 1 et être renormalisable. À un niveau non-perturbatif, des phénomènes spectaculaires sont susceptibles de se produire. L'un d'eux est le confinement des quarks, attendu en QCD. De plus, il existe dans nos théories, comme dans le modèle de Georgi-Glashow, des monopôles magnétiques et des dyons (particules à la fois chargées électriquement et magnétiquement), en plus des particules habituelles du spectre perturbatif (photon, bosons W, etc...). Malgré l'origine très différente de toutes ces particules, il existe une formule remarquable, que l'on peut déduire facilement dans le modèle de Georgi-Glashow à un niveau semi-classique, qui donne la masse d'une particule quelconque du modèle en fonction de la valeur moyenne du Higgs, de la charge électrique Q_e et de la charge magnétique Q_m:

$$M = |\langle\phi\rangle|\,\sqrt{Q_e^2+Q_m^2}. \tag{1}$$

Cette formule, invariante par échange de Q_e et Q_m, suggère qu'une équivalence entre particules chargées électriquement et particules chargées magnétiquement pourrait exister [1]. Une telle équivalence, si elle était correcte, aurait des conséquences remarquables. En effet, si g est la constante de couplage de jauge de notre théorie, la charge électrique Q_e est bien sûr proportionnelle à g, mais la charge magnétique Q_m doit être proportionnelle à $1/g$, un résultat dû à Dirac. Ainsi, si le couplage électrique g est grand (comme en QCD), et que les développements perturbatifs habituels ne sont pas valables, il est envisageable de considérer une formulation de la théorie en termes de particules chargées magnétiquement, qui elles ont un couplage en $1/g$ qui est faible et en terme duquel on peut donc écrire des développements perturbatifs.

Comme on peut s'y attendre, une telle image ne peut être correcte en toute généralité. Tout d'abord, la formule fondamentale (1) ne résiste pas aux corrections quantiques, *à moins de considérer des théories ayant au moins deux supersymétries.* De plus, une équivalence quantique complète entre deux théories de jauge ayant des couplages inverses ne peut être vraie que si la fonction β est nulle dans les deux théories, ce qui limite fortement les possibilités. Notons tout de même que de tels cas idéaux existent et ont un intérêt théorique indéniable. Néanmoins, dans la plupart des cas, on peut au mieux espérer que la dualité électrique-magnétique soit vraie de manière approchée, dans certaines limites, comme au niveau de l'action effective à basse énergie. Par exemple, lorsque le groupe de jauge est brisé en $\mathrm{U}(1)^r$, l'action effective est une théorie de jauge *abélienne* dont la constante de couplage g_{eff} tend vers zero dans l'infrarouge et est donc susceptible d'être reliée à la constante de couplage g de la théorie microscopique asymptotiquement libre par une relation du type $g_{\text{eff}} \sim 1/g$.

Comment donc, au-delà de ces considérations qualitatives, décider si une théorie a une chance d'être duale d'une autre (au moins dans certaines limites)? On peut montrer qu'une telle équivalence à des conséquences hautement non-triviales sur le spectre des particules d'une théorie donnée. Par exemple, les dyons susceptibles d'être échangés avec les particules chargées électriquement doivent, au-delà de leur masse, avoir des nombres quantiques (spin, saveur, . . .) compatibles. Ceci est bien possible dans le cadre des théories supersymétriques. D'autre part, ces dyons sont en général des *états liés* de dyons élémentaires. L'existence même de ces états liés au niveau quantique est loin d'être évidente. Pour l'établir, il faut mettre au point une théorie quantique des monopôles (qui est une mécanique quantique supersymétrique), dont l'étude constitue un problème mathématique très complexe. La difficulté de ces problèmes est telle que lorsque Sen montra en 1994 l'existence d'états liés à deux monopôles dans la théorie la plus simple (ayant quatre supersymétries) [2], alors que la dualité implique en fait l'existence d'états liés avec un nombre quelconque de monopôles, il se déclencha un véritable engouement pour ces sujets, qui ne s'est pas éteint depuis, et dont l'un des achèvements les plus remarquables reste le détermination exacte par Seiberg et Witten de l'action effective à basse énergie dans certains modèles [3].

Il est important de noter que les développements récents, s'ils sont confinés (pour l'instant?) aux théories supersymétriques, ont d'ores et déjà jeté une lumière nouvelle sur la théorie des champs et en particulier sur les phénomènes attachés à la dynamique des champs de jauge en couplage fort. Je terminerai cet exposé en illustrant ceci par trois exemples. Le premier concerne le statut de la particule élémentaire. Depuis toujours, la particule élémentaire a été définie comme étant un constituant *sans structure interne.* Or, j'ai souligné cidessus que la dualité prédisait l'équivalence entre une formulation des théories en termes des particules élémentaires habituelles (chargées électriquement) et une formulation en termes de monopôles magnétiques ou de dyons *qui apparaissent dans la formulation initiale de la théorie comme étant des états liés.* Ainsi, le statut de particule sans structure interne ou composite est-il relatif, et dépend du point de vue (de la formulation de la théorie) adopté. Ceci constitue à mes yeux une révolution conceptuelle de grande importance. Le deuxième exemple concerne le statut des bosons de jauge massifs. À la brisure spontanée de la symétrie de jauge est associée habituellement la présence de particules massives de spin 1, les "bosons W:" c'est le mécanisme de Higgs. Cependant, la preuve de l'existence de ces bosons W est de nature perturbative (contrairement, par exemple, au théorème de Goldstone), et sa validité pour des théories fortement couplées peut être questionnée. En fait, il a été démontré pour la première fois dans [4] que dans certains régimes de la QCD supersymétrique, aucune particule massive de spin 1 n'était associée à la brisure de la symétrie de jauge SU(2) en U(1). Le spectre de la théorie est en fait complètement différent du contenu en champ dans le lagrangien, ce qui montre qu'une formulation lagrangienne habituelle n'est pas du tout adaptée à la physique du couplage fort. Pourra-t-on trouver une formulation mieux adaptée? Ceci nous amène à mon troisième et dernier exemple, qui se situe à un niveau plus spéculatif, et qui concerne le lien entre la théorie des cordes et les théories de jauge. Il est apparu que certains des aspects curieux de la physique des champs de jauge en couplage fort avaient une interprétation très naturelle en théorie des cordes. Cette théorie, au-delà de son intérêt intrinsèque en tant que théorie candidate de la gravité quantique, pourrait fournir un cadre adapté à l'étude des phénomènes non-perturbatifs en théorie des champs, un peu comme les théories conformes ont permis de comprendre les phénomènes critiques bidimensionnels [5].

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Higgs Mechanism

When a local rather than global symmetry is spontaneously broken, we do not get a massless Goldstone boson. Instead, the gauge field of the broken symmetry becomes massive, and the would-be Goldstone scalar becomes the longitudinal mode of the massive vector. This is the *Higgs mechanism*, and it works for both abelian and non-abelian local symmetries. In the non-abelian case, for each spontaneously broken generator T^a of the local symmetry the corresponding gauge field $A^a_\mu(x)$ becomes massive.

The Abelian Example

To understand how the Higgs mechanism works, let's start with the abelian example of a local $U(1)$ phase symmetry. The complete model comprises a complex scalar field $\Phi(x)$ of electric charge q coupled to the EM field $A^\mu(x)$; the Lagrangian is

$$\mathcal{L} \;=\; -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \;+\; D_\mu\Phi^* D^\mu\Phi \;-\; V(\Phi^*\Phi) \tag{1}$$

where

$$D_\mu\Phi(x) \;=\; \partial_\mu\Phi(x) \;+\; iqA_\mu(x)\Phi(x), \qquad D_\mu\Phi^*(x) \;=\; \partial_\mu\Phi^*(x) \;-\; iqA_\mu(x)\Phi^*(x), \tag{2}$$

and

$$V(\Phi^*\Phi) \;=\; \frac{\lambda}{2}(\Phi^*\Phi)^2 \;+\; m^2(\Phi^*\Phi). \tag{3}$$

Suppose $\lambda > 0$ but $m^2 < 0$, so that $\Phi = 0$ is a local maximum of the scalar potential, while the minima form a degenerate circle

$$\Phi \;=\; \frac{v}{\sqrt{2}} \times e^{i\theta}, \quad v \;=\; \sqrt{\frac{-2m^2}{\lambda}}, \quad \text{any real } \theta. \tag{4}$$

Consequently, the scalar field Φ develops a non-zero vacuum expectation value $\langle\Phi\rangle \neq 0$, which spontaneously breaks the $U(1)$ symmetry of the theory. Were that $U(1)$ symmetry global rather than local, is spontaneous breakdown would lead to a massless Goldstone scalar stemming from the phase of the complex field $\Phi(x)$. But for the local $U(1)$ symmetry, the phase of $\Phi(x)$ — not just the phase of the vacuum expectation value $\langle\Phi\rangle$ but the x-dependent phase of the dynamical $\Phi(x)$ field — can be eliminated by a gauge transform, so the physical consequences of the SSB are more complicated.

To see how this works, let's use polar coordinates in the scalar field space, thus

$$\Phi(x) \;=\; \frac{1}{\sqrt{2}}\,\phi_r(x)\times e^{i\Theta(x)}, \quad \text{real } \phi_r(x)>0, \quad \text{real } \Theta(x). \tag{5}$$

This field redefinition is singular when $\Phi(x)=0$, so we should never use it for theories with $\langle\Phi\rangle=0$, but it's OK for spontaneously broken theories where we expect $\Phi(x)\neq 0$ almost everywhere. In terms of the real fields $\phi_r(x)$ and $\Theta(x)$, the scalar potential depends only on the radial field ϕ_r,

$$V(\Phi) \;=\; \frac{\lambda}{8}\big(\phi_r^2-v^2\big)^2 \;+\; \text{const}, \tag{6}$$

or in terms of the radial field shifted by its VEV, $\phi_r(x)=v+\sigma(x)$,

$$\phi_r^2 \;-\; v^2 \;=\; (v+\sigma)^2 \;-\; v^2 \;=\; 2v\sigma \;+\; \sigma^2, \tag{7}$$

$$V \;=\; \frac{\lambda}{8}\big(2v\sigma+\sigma^2\big)^2 \;=\; \frac{\lambda v^2}{2}\times\sigma^2 \;+\; \frac{\lambda v}{2}\times\sigma^3 \;+\; \frac{\lambda}{8}\times\sigma^4. \tag{8}$$

At the same time, the covariant derivative $D_\mu\Phi$ becomes

$$D_\mu\Phi \;=\; \frac{1}{\sqrt{2}}\Big(\partial_\mu\big(\phi_r e^{i\Theta}\big) \;+\; iqA_\mu\times\phi_r e^{i\Theta}\Big) \;=\; \frac{e^{i\Theta}}{\sqrt{2}}\Big(\partial_\mu\phi_r \;+\; \phi_r\times i\partial_\mu\Theta \;+\; \phi_r\times iqA_\mu\Big). \tag{9}$$

Inside the big () on the RHS, the first term is real while the other two terms are imaginary, hence

$$\begin{aligned}
|D_\mu\Phi|^2 \;&=\; \frac{1}{2}\Big|\partial_\mu\phi_r \;+\; \phi_r\times i\partial_\mu\Theta \;+\; \phi_r\times iqA_\mu\Big|^2 \\
&=\; \tfrac{1}{2}(\partial_\mu\phi_r)^2 \;+\; \frac{\phi_r^2}{2}\times\big(\partial_\mu\Theta+qA_\mu\big)^2 \\
&=\; \tfrac{1}{2}(\partial_\mu\sigma)^2 \;+\; \frac{(v+\sigma)^2}{2}\times\big(\partial_\mu\Theta+qA_\mu\big)^2.
\end{aligned} \tag{10}$$

Altogether,

$$\mathcal{L} \;=\; \tfrac{1}{2}(\partial_\mu\sigma)^2 \;-\; V(\sigma) \;-\; \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \;+\; \frac{(v+\sigma)^2}{2}\times\big(\partial_\mu\Theta+qA_\mu\big)^2. \tag{11}$$

To understand the physical content of this Lagrangian, let's expand it in powers of the

fields (and their derivatives) and focus on the quadratic part describing the free particles,

$$\mathcal{L}_{\text{free}} \;=\; \tfrac{1}{2}(\partial_\mu\sigma)^2 \;-\; \frac{\lambda v^2}{2}\times\sigma^2 \;-\; \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \;+\; \frac{v^2}{2}\times\bigl(qA_\mu \;+\; \partial_\mu\Theta\bigr)^2. \tag{12}$$

The first two terms here obviously describe a real scalar particle of positive mass$^2 = \lambda v^2$. The other two terms — involving the $A_\mu(x)$ and the $\Theta(x)$ fields — seem to describe a photon and a scalar field, but in fact describe a massive vector field and no scalars!

To see how this works, note the *local* $U(1)$ symmetry of the theory, which acts as

$$\begin{aligned} A'_\mu(x) \;&=\; A_\mu(x) \;-\; \partial\Lambda(x), \\ \Phi'(x) \;&=\; \Phi(x)\times\exp(iq\Lambda(x)), \\ \sigma'(x) \;&=\; \sigma(x), \\ \Theta'(x) \;&=\; \Theta(x) \;+\; q\Lambda(x), \end{aligned} \tag{13}$$

for an arbitrary x-dependent $\Lambda(x)$. Physically, such a local symmetry means that one of the 6 field variables at each x — the real and the imaginary parts of the $\Phi(x)$, and the 4 components of the $A^\mu(x)$ — is redundant, and we may reduce this redundancy by imposing a gauge-fixing condition such as the Coulomb gauge $\nabla\cdot\mathbf{A}(x)\equiv 0$ or the Landau gauge $\partial_\mu A^\mu(x)\equiv 0$. When we have a charged scalar field with a non-zero VEV, we may also impose a gauge-fixing condition on that scalar field (instead of the vector field $A^\mu(x)$), thus *the unitary gauge*

$$\Theta(x) \;=\; \text{phase}\bigl(\Phi(x)\bigr) \;\equiv\; 0. \tag{14}$$

The unitary gauge is badly singular when the complex field $\Phi(x)$ fluctuates around zero, so it should never be used for the gauge symmetries which are NOT spontaneously broken. But when the symmetry IS spontaneously broken by $\langle\Phi\rangle\neq 0$ and the points where $\Phi(x)$ vanishes are few and far between (if they exist at all), the phase $\Theta(x)$ is well-defined almost everywhere, and it is easy to gauge it away by setting $\Lambda(x) = (-1/q)\Theta(x) \implies \Theta'(x) = 0$.

In the unitary gauge, the last two terms in the free Lagrangian (12) become simply

$$\mathcal{L}^{\text{massive}}_{\text{vector}} \;=\; -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \;+\; \frac{q^2v^2}{2}\times A_\mu A^\mu, \tag{15}$$

the Lagrangian of a massive vector field of mass $m_v = qv$. The scalar $\Theta(x)$ is gone from this Lagrangian — it was eliminated by the unitary gauge fixing. For the same reason, the

Lagrangian (11) is NOT gauge invariant — we used up the gauge symmetry of the original theory for eliminating the $\Theta(x)$ field, and now the remaining $A^\mu(x)$ field does not have any gauge symmetry anymore.

Without the unitary gauge — or any other gauge-fixing condition — we may describe exactly the same massive vector particles using *redundant fields* $A^\mu(x)$ and $\Theta(x)$ subject to gauge symmetry

$$A'_\mu(x) \;=\; A_\mu(x) \;-\; \partial_\mu\Lambda(x), \quad \Theta'(x) \;=\; \Theta(x) \;+\; q\Lambda(x), \tag{16}$$

and a gauge-invariant free Lagrangian

$$\mathcal{L}^{\text{massive}}_{\text{vector}} \;=\; -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \;+\; \frac{(qv)^2}{2}\times\bigl(A_\mu \,+\, q^{-1}\partial_\mu\Theta\bigr)^2. \tag{17}$$

But the $\Theta(x)$ field here is not physical, it does not give rise to any scalar particles, and its plane waves are mere gauge artefacts. The only physical particles in this system are the massive vector particles, the same as in the Θ-less unitary-gauge Lagrangian (15).

Altogether, the complete particle spectrum of the theory of $\Phi(x)$ and $A^\mu(x)$ fields with a spontaneously-broken local $U(1)$ symmetry comprises a massive real scalar $\sigma(x)$ and a massive vector. But there is NO massless Goldstone scalar!

To see what happened to the would-be Goldstone boson, let's count the degrees of freedom of the complete theory. The complex scalar field $\Phi(x)$ carries 2 degrees of freedom, while the vector field $A_\mu(x)$ subject to gauge symmetry carries another 2 DoF, for the total of 4 DoF. This means that for every momentum 3-vector $\mathbf{k}$, there should be 4 distinct 1-particle states $|\mathbf{k}, ??\rangle$ belonging to different particle species or different spin/polarization states. This counting should work for both spontaneously-broken or unbroken $U(1)$ symmetry, although the specific 1-particle states turn out to be quite different for the two regimes:

- The unbroken $U(1)$ regime for $m^2 > 0$ and $\langle\Phi\rangle = 0$:
  In this regime, the $A^\mu(x)$ fields describe a massless photon, which has 2 helicity states, $\lambda = \pm 1$ (but not $\lambda = 0$). At the same time, the complex scalar field $\Phi(x)$ with an unbroken $U(1)$ symmetry describes 2 scalar particle species with opposite electric charges $\pm q$, the particle and the antiparticle. Altogether, for each $\mathbf{k}$ there are 4 1-particle states:

the scalar particle $\left|S^+\right\rangle$, the antiparticle $\left|S^-\right\rangle$, and two photon states $|\gamma(\lambda =)\rangle$ and $|\gamma(\lambda = -1)\rangle$.

- The spontaneously-broken $U(1)$ regime for $m^2 < 0$ and $\langle\Phi\rangle \neq 0$:
  In this regime, there is only one scalar particle species σ, but the massive vector particle has 3 spin states, $\lambda = -1, 0, +1$. Again, altogether there are 4 1-particle states: the $|\sigma\rangle$, and the $|V(\lambda = +1)\rangle$, $|V(\lambda = 0)\rangle$, $|V(\lambda = -1)\rangle$.
  - ⋆ But these are rather different 4 states from the unbroken $U(1)$ regime!

Now we can see what happens in the spontaneously-broken regime to the would-be Goldstone boson $\Theta(x)$: It became the longitudinal $\lambda = 0$ polarization of the massive vector field! Indeed, the unbroken-symmetry regime has a massless vector without the $\lambda = 0$ mode. Once the symmetry is spontaneously broken and the vector becomes massive, it has to have all 3 spin states, including the $\lambda = 0$ longitudinal mode. That mode has to come from somewhere, so the Higgs mechanism 'eats up' the would-be Goldstone scalar $\Theta(x)$ and turns it into the longitudinal polarization of the massive vector!

A rigorous way to see how this works would be to start with the redundant gauge-invariant description (16) or a massive vector field, fix the Coulomb gauge $\nabla\cdot\mathbf{A} = 0$ instead of the unitary gauge, expand the Lagrangian (17) into Fourier and helicity modes, eliminate the modes of the A^0 field, quantize the theory canonically, and in the process see how the $\hat{\Theta}_{\mathbf{k}}$ and the $\hat{\Pi}^{\Theta}_{\mathbf{k}}$ operators combine into the creation and annihilation operators for the longitudinally polarized vector particles. But this is a lot of work, and I am not going to do it here. Instead, I let the unitary gauge speak for the outcome of the Higgs mechanism, even if it hides the gory details of the 'eating up the Goldstone boson'.

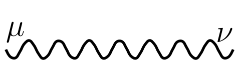
To complete this section, let me write down the complete Lagrangian of the spontaneously-broken theory in the unitary gauge, including all the interactions of the $\sigma(x)$ fields with itself and with the massive vector field:

$$\begin{aligned}\mathcal{L} \;=\; & \tfrac{1}{2}(\partial_\mu\sigma)^2 \;-\; \frac{\lambda v^2}{2}\times\sigma^2 \;-\; \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \;+\; \frac{q^2v^2}{2}\times A_\mu A^\mu \\ & -\; \frac{\lambda v}{2}\times\sigma^3 \;-\; \frac{\lambda}{8}\times\sigma^4 \;+\; qv^2\times\sigma\,A_\mu A^\mu \;+\; \frac{q^2}{2}\times\sigma^2\,A_\mu A^\mu.\end{aligned} \tag{18}$$

Propagators and Gauges for Massive Vectors

The unitary gauge is very useful for identifying the physical particles of the theory. This will be particularly convenient in the Higgsed non-abelian gauge theories, as we shall see later in these notes. But in perturbation theory, the unitary gauge is useful at the tree level but in loop diagrams it makes the UV divergences much worse than in the massless theory. To avoid this problem, perturbation theory beyond the tree level is usually done using non-unitary Feynman-like R_ξ gauges I shall explain in a minute.

To see the problem with loop diagrams, consider the massive photon's propagator. In the unitary gauge, the massive photon is simply an ordinary massive vector field with free Lagrangian (15), so its propagator is just the massive vector propagator from homework#4,



$$= \frac{i}{k^2 - m^2 + i0}\left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{m^2}\right) \quad \text{for } m = qv. \tag{19}$$

Note that at large off-shell momenta $k \gg m$ this propagator behaves as $O(1/m^2)$, unlike the massless photon's propagator which behaves as $O(1/k^2)$. So when the propagator (19) is a part of some loop and we integrate over the loop momentum, we get

$$\int\frac{d^4k}{(2\pi)^4}\begin{pmatrix}\text{massive photon}\\ \text{propagator}\end{pmatrix} \times \begin{pmatrix}\text{other propagators}\\ \text{and vertices}\end{pmatrix}, \tag{20}$$

the integrand decreases for $k^\mu \to \infty$ much slower than for a similar diagram involving a massless photon. Consequently, the integral (20) generally suffers from much worse ultraviolet divergence — the divergence for $k \to \infty$ — than a similar integral in a massless QED. In the QFT (II) class we shall learn how handle the UV divergences in QED and other *renormalizable* theories. The worse divergences due to the massive photon propagator (19) in the unitary gauge would break the renormalization techniques and make the massive theory non-renormalizable. In other words, it would be a good effective low-energy theory for the tree-level calculations but unsuitable for the loop calculations.

Fortunately, the problem is not with the massive photons *per se* but rather with the unitary gauge. Other gauges exist where the massive photon propagator behaves like $O(1/k^2)$ for large off-shell momenta, and in those gauges the UV divergences of the massive theory are similar to its massless counterpart.

Indeed, consider a non-unitary gauge in which the would-be Goldstone scalar $\pi(x) = \operatorname{Im}\Phi(s)$ is not restricted at all; instead, we impose a Landau gauge condition $\partial_\mu A^\mu(x) \equiv 0$ on the vector field itself. Or rather, impose a Landau-like gauge condition

$$\partial_\mu A^\mu(x) \;-\; \xi m\pi(x) \;=\; \text{given } f(x), \tag{21}$$

and then in the path-integral formulation of the theory (which we shall learn in the QFT II class), we *average* over all possible $f(x)$. The result of such gauge-averaging is equivalent to not imposing any gauge condition at all but rather adding a gauge-fixing

$$\Delta\mathcal{L} \;=\; -\frac{1}{2\xi}\Big(\partial_\mu A^\mu \;-\; \xi m\pi\Big)^2 \tag{22}$$

to the Lagrangian of the theory. For a massless theory (no Higgs mechanism), the net quadratic Lagrangian

$$\mathcal{L}_2 \;=\; -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \;+\; \Delta\mathcal{L} \tag{23}$$

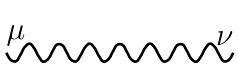
yields the Feynman gauge (or Feynman-like gauges for $\xi \neq 1$) for the photon propagator

$$= \; \frac{i}{k^2 + i0}\left(-g^{\mu\nu} \;+\; (1-\xi)\frac{k^\mu k^\nu}{k^2 + i0}\right). \tag{24}$$

For the massive photon due to Higgs mechanism, we combine the vector field $A^\mu(x)$ and the would-be Goldstone scalar $\pi(x)$ with free Lagrangian

$$\begin{aligned} \mathcal{L}_2 \;&=\; -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \;+\; \tfrac{1}{2}(\partial_\mu\pi)^2 \;+\; \Delta\mathcal{L} \\ &=\; -\tfrac{1}{2}(\partial_\mu A_\nu)^2 \;+\; \tfrac{1}{2}(1-\xi^{-1})(\partial_\lambda A^\lambda)^2 \;+\; \tfrac{1}{2}m^2 A_\mu A^\mu \;+\; \tfrac{1}{2}(\partial_\mu\pi)^2 \;-\; \tfrac{1}{2}\xi m^2\pi^2. \end{aligned} \tag{25}$$

Consequently, we get the so-called R_ξ gauge in which the massive vector field has propagator



$$= \; \frac{i}{k^2 - m^2 + i0}\left(-g^{\mu\nu} \;+\; (1-\xi)\frac{k^\mu k^\nu}{k^2 - \xi m^2 + i0}\right) \;\xrightarrow[\xi=1]{}\; \frac{-ig^{\mu\nu}}{k^2 - m^2 + i0}. \tag{26}$$

Unlike the unitary gauge propagator (19), this propagator behaves as $1/k^2$ for large off-shell momenta k, so the renormalization works similar to QED. (Although the technical aspects of renormalization are much more complicated in the Higgsed case, and it took much longer to prove the theory's renormalizability.)

The price of a non-unitary gauge like R_ξ is that the unphysical would-be Goldstone field $\pi(x)$ is not eliminated for the theory. Instead, it remains in the theory, couples to the physical Higgs scalar $\sigma(x)$, and perhaps to the other fields (if the theory has any). In Feynman rules, the $\pi(x)$ has a propagator

$$\cdots\cdots\cdots\cdots\cdots\cdot \;=\; \frac{i}{k^2 - \xi m^2 + i0}\,. \tag{27}$$

Note the pole at $k^2 = \xi m^2$, which is not the physical mass^2 of any particle unless $\xi = 1$; this spurious pole is needed to cancel the other unphysical effects of the massive photon propagator (26).

To summarize this section, loop calculations requires gauges like R_ξ where the Feynman rules involve both physical and unphysical fields. However, the tree-level calculation can also be done in the unitary gauge which involves only the physical fields. And the semi-classical calculations where we identify the particles and find their masses — as we shall do in a moment for the non-abelian Higgs mechanism — are best done in the unitary gauge.

Non-Abelian Higgs Mechanism

Example: SU(2) with a Higgs Doublet

To illustrate the non-abelian Higgs mechanism, consider the example of $SU(2)$ gauge theory coupled to a doublet of complex scalar fields $\Phi^i(x)$. In terms of canonically normalized fields, the Lagrangian is

$$\mathcal{L} \;=\; -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} \;+\; D_\mu\Phi^*_i D^\mu\Phi^i \;-\; \frac{\lambda}{2}\left(\Phi^*_i\Phi^i \;-\; \frac{v^2}{2}\right)^2, \tag{28}$$

where

$$\begin{aligned} D_\mu\Phi^i \;&=\; \partial_\mu\Phi^i \;+\; \tfrac{i}{2}gA^a_\mu\big(\sigma^a\big)^i{}_j\Phi^j\,, \\ D_\mu\Phi^*_i \;&=\; \partial_\mu\Phi^*_i \;-\; \tfrac{i}{2}gA^a_\mu\Phi^*_j\big(\sigma^a\big)^j{}_i\,, \\ F^a_{\mu\nu} \;&=\; \partial_\mu A^a_\nu \;-\; \partial_\nu A^a_\mu \;-\; g\epsilon^{abc}A^b_\mu A^c_\nu\,. \end{aligned} \tag{29}$$

For $v^2 > 0$ the scalar potential has a local maximum at $\Phi^i = 0$ while the minima form a spherical shell $\Phi^*_i\Phi^i = (v^2/2)$ in the $\mathbf{C}^2 = \mathbf{R}^4$ field space; all such minima are related by the

$SU(2)$ symmetries to

$$\langle\Phi\rangle = \frac{v}{\sqrt{2}} \times \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{30}$$

Note that this vacuum expectation value spontaneously breaks the $SU(2)$ symmetry down to nothing — there is no subgroup of $SU(2)$ which leaves this VEV invariant. Consequently, we expect all 3 vector fields $A^a_\mu(x)$ to become massive.

In the process, 3 would-be Goldstone scalars should be eaten by the Higgs mechanism. Since the theory has 2 complex — or equivalently 4 real — scalars, only one real scalar should survive un-eaten. Ironically, it is this un-eaten scalar $\sigma(x)$ which is called *the physical Higgs field.*

To see how this works, let's fix the unitary gauge. Any complex doublet $\Phi^i(x)$ can be rotated by some $SU(2)$ symmetry $U(x)$ so that the upper component of the rotated $\Phi' = U\Phi$ is zero, $\Phi'^1 = 0$, while the lower component Φ'^2 is real and positive. Thus, in the unitary gauge we require

$$\begin{aligned} \operatorname{Re}\Phi^1(x) \;\equiv\; \operatorname{Im}\Phi^1(x) \;\equiv\; \operatorname{Im}\Phi^2(x) \;\equiv\; 0, \\ \text{hence}\quad \Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \phi_r(x) \end{pmatrix} \quad \text{for a real } \phi_r(x) > 0. \end{aligned} \tag{31}$$

This gauge-fixing condition is terribly singular for $\phi_r \to 0$, so it should never be used for the unbroken-symmetry regime of the theory. But for the spontaneously broken theory where $\phi_r(x)$ fluctuates around the minimum at $\phi_r = v > 0$, the unitary gauge is OK.

In the unitary gauge, the only scalar field is the $\phi_r(x)$, or equivalently the shifted field $\sigma(x) = \phi_r(x) - v$; all the other scalar fields are frozen by the gauge-fixing conditions (31). In terms of physical Higgs field $\sigma(x)$, the scalar potential becomes

$$V = \frac{\lambda}{2}\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2 = \frac{\lambda}{8}\left(2v\sigma + \sigma^2\right)^2 = \frac{\lambda v^2}{2} \times \sigma^2 + \frac{\lambda v}{2} \times \sigma^3 + \frac{\lambda}{8} \times \sigma^4 \tag{32}$$

where the first terms is the mass term, $\text{mass}^2 = \lambda v^2$, while the remaining terms are self-

interactions. More interestingly, the covariant derivative of the Higgs doublet Φ becomes

$$
\begin{aligned}
D_\mu\Phi \;&=\; \frac{1}{\sqrt{2}}\Bigg[\begin{pmatrix} 0 \\ \partial_\mu\sigma \end{pmatrix} + \frac{ig}{2}A^3_\mu \times \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ v+\sigma \end{pmatrix} \\
&\qquad + \frac{ig}{2}A^1_\mu \times \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v+\sigma \end{pmatrix} + \frac{ig}{2}A^2_\mu \times \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v+\sigma \end{pmatrix}\Bigg] \\
&=\; \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{i}{2}g\big(A^1_\mu - iA^2_\mu\big)\times(v+\sigma) \\ \partial_\mu\sigma \;-\; \frac{i}{2}g\,A^3_\mu \times (v+\sigma) \end{pmatrix},
\end{aligned}
\tag{33}
$$

hence

$$
\begin{aligned}
D_\mu\Phi^\dagger\, D^\mu\Phi \;&=\; \tfrac{1}{2}\left|\tfrac{i}{2}g\big(A^1_\mu - iA^2_\mu\big)\times(v+\sigma)\right|^2 \;+\; \tfrac{1}{2}\left|\partial_\mu\sigma \;-\; \tfrac{i}{2}g\,A^3_\mu\times(v+\sigma)\right|^2 \\
&=\; \frac{g^2(v+\sigma)^2}{8}\times\Big(\big(A^1_\mu\big)^2 \;+\; \big(A^2_\mu\big)^2\Big) \;+\; \frac{g^2(v+\sigma)^2}{8}\times\big(A^3_\mu\big)^2 \;+\; \tfrac{1}{2}(\partial_\mu\sigma)^2.
\end{aligned}
\tag{34}
$$

The last term here is the kinetic term for the Higgs scalar $\sigma(x)$, while the rest of the bottom line are mass terms for the vector fields and the interaction terms between the vectors and the σ. Curiously, we get the same mass and similar interactions for all 3 vector fields A^a_μ:

$$
\mathcal{L} \;\supset\; \frac{g^2(v+\sigma)^2}{8}\,A^a_\mu A^{a\mu} \;=\; \frac{M^2}{2}\times A^a_\mu A^{a\mu} \;+\; \frac{g^2 v}{4}\times\sigma A^a_\mu A^{a\mu} \;+\; \frac{g^2}{8}\times\sigma^2 A^a_\mu A^{a\mu}
\tag{35}
$$

where

$$
M^2 \;=\; \frac{g^2v^2}{4}\,.
\tag{36}
$$

Example: SU(2) with a Higgs Triplet

Now consider an example of a partially broken gauge symmetry, an $SU(2)$ Higgsed down to a $U(1)$ subgroup, or equivalently $SO(3)\to SO(2)$. This time, the scalar fields $\Phi^a(x)$ are real and form a triplet of the $SU(2)$ rather than a doublet. Thus,

$$
\mathcal{L} \;=\; -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} \;+\; \tfrac{1}{2}D_\mu\Phi^a D^\mu\Phi^a \;-\; \frac{\lambda}{8}\left(\Phi^a\Phi^a \;-\; v^2\right)^2,
\tag{37}
$$

where

$$
D_\mu\Phi^a \;=\; \partial_\mu\Phi^a \;-\; g\epsilon^{abc}A^b_\mu\Phi^c\,, \qquad F^a_{\mu\nu} \;=\; \partial_\mu A^a_\nu \;-\; \partial_\nu A^a_\mu \;-\; g\epsilon^{abc}A^b_\mu A^c_\nu\,.
\tag{38}
$$

Again, for $v^2>0$ the scalar potential $V(\Phi)$ has a degenerate family of minima which form a

spherical shell $\Phi^a\Phi^a = v^2$ in the scalar field space $\mathbf{R}^3$, and all such minima are equivalent by $SU(2) \cong SO(3)$ symmetries to

$$\langle\Phi\rangle \;=\; \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}. \tag{39}$$

This time, this vacuum expectation value is invariant under an $SO(2)$ subgroup of the $SO(3)$, — or equivalently under an $U(1)$ subgroup of the $SU(2)$. Specifically, it's the $SO(2) \cong U(1)$ generated by the T^3, the third component of the isospin $\mathbf{T}$. Consequently, out of the 3 vector fields A^a_μ, we expect the A^3_μ to remain massless while the other 2 fields $A^{1,2}_\mu$ should become massive.

In the process, the Higgs mechanism should eat 2 real scalar fields. Since we only have 3 real scalars to begin with, only one scalar should survive un-eaten — the Physical Higgs field $\sigma(x)$.

To see how this works, we fix the unitary gauge

$$\Phi^1(x) \;\equiv\; \Phi^2(x) \;\equiv\; 0, \qquad \Phi^3(x) \;>\; 0. \tag{40}$$

As usual, this gauge is badly singular at $\Phi = 0$, but it's OK for the $\Phi(x) \approx \langle\Phi\rangle \neq 0$. Shifting the $\Phi^3(x)$ by the VEV, we get $\Phi^3(x) = v + \sigma(x)$, where $\sigma(x)$ is the physical Higgs scalar — and also the only scalar remaining in the theory in the unitary gauge.

In terms of the $\sigma(x)$, the scalar potential becomes

$$V(\sigma) \;=\; \frac{\lambda}{8}\left(2v\sigma + \sigma^2\right)^2 \;=\; \frac{\lambda v^2}{2}\times\sigma^2 + \frac{\lambda v}{2}\times\sigma^3 \;+\; \frac{\lambda}{8}\times\sigma^4, \tag{41}$$

where the first terms on the RHS gives the Higgs scalar mass$^2 = \lambda v^2$. More interestingly, the

covariant derivative of the scalar triple $\Phi^a(x)$ becomes

$$D_\mu \Phi^a = \begin{pmatrix} 0 \\ 0 \\ \partial_\mu \sigma \end{pmatrix} - g \begin{pmatrix} A_\mu^1 \\ A_\mu^2 \\ A_\mu^3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ v+\sigma \end{pmatrix}$$

$$\langle\!\langle \text{ where } \times \text{ is the cross product of two isovectors } \rangle\!\rangle \tag{42}$$

$$= \begin{pmatrix} -gA_\mu^2(v+\sigma) \\ +gA_\mu^1(v+\sigma) \\ \partial_\mu \sigma \end{pmatrix},$$

hence the covariant kinetic terms for the scalars become

$$\tfrac{1}{2} D_\mu \Phi^a D^\mu \Phi^a = \tfrac{1}{2}(\partial_\mu \sigma)^2 + \frac{g^2(v+\sigma)^2}{2} \times \Big(\big(A_\mu^1\big)^2 + \big(A_\mu^2\big)^2 \Big). \tag{43}$$

As usual, the first term here is the kinetic term for the physical Higgs scalar σ, while the second term contains the mass terms for the vector fields,

$$\mathcal{L} \supset \frac{M^2}{2} \times \Big(\big(A_\mu^1\big)^2 + \big(A_\mu^2\big)^2 \Big), \quad M^2 = g^2 v^2, \tag{44}$$

but only for the A_μ^1 and the A_μ^2 — the third vector $A_\mu^3(x)$ remains massless.

The massless vector $A_\mu^3(x)$ is the gauge field of the un-Higgsed $SO(2) \cong U(1)$ subgroup of the $SO(3) \cong SU(2)$. Interpreting this gauge field as the EM field and hence the rescaled generator $Q = gT^3$ as the electric charge operator, we find that the physical Higgs field is electrically neutral while the massive vector fields have electric charges $q = \pm g$. To be precise, the massive vector fields of definite charges are not the A_μ^1 and the A_μ^2 themselves but rather their linear combination

$$W_\mu^+ = \frac{1}{\sqrt{2}}\big(A_\mu^1 - iA_\mu^2\big) \quad \text{and} \quad W_\mu^- = \frac{1}{\sqrt{2}}\big(A_\mu^1 + iA_\mu^2\big) \quad \text{of charges } q = \pm g. \tag{45}$$

For completeness sake, let's re-express the theory at hand (usually called the *Georgi–Glashow*

model) in terms of the physical fields of definite charges. Using $U(1)$–covariant derivatives

$$\widetilde{D}_\mu W^\pm_\nu \;=\; \partial_\mu W^\pm_\nu \;\pm\; igA^3_\mu W^\pm_\nu\,, \tag{46}$$

we have

$$W^\pm_{\mu\nu} \;\stackrel{\rm def}{=}\; \frac{1}{\sqrt{2}}\bigl(F^1_{\mu\nu} \mp iF^\pm_{\mu\nu}\bigr) \;=\; \widetilde{D}_\mu W^\pm_\nu \;-\; \widetilde{D}_\nu W^\pm_\mu\,, \tag{47}$$

but

$$F^3_{\mu\nu} \;=\; \widetilde{F}_{\mu\nu} \;+\; 2g\,\mathrm{Im}\bigl(W^+_\mu W^-_\nu\bigr) \quad\text{where}\quad \widetilde{F}_{\mu\nu} \;=\; \partial_\mu A^3_\nu \;-\; \partial_\nu A^3_\mu\,. \tag{48}$$

Consequently, the Lagrangian of the whole model — the kinetic terms, the mass terms, and the interactions — can be expressed as

$$\begin{aligned}\mathcal{L} \;=&\; \tfrac12(\partial_\mu\sigma)^2 \;-\; \tfrac12 M^2_\sigma\times\sigma^2 \;-\; \tfrac14\widetilde{F}_{\mu\nu}\widetilde{F}^{\mu\nu} \;-\; \tfrac12 W^+_{\mu\nu}W^{-\mu\nu} \;+\; M^2_W W^+_\mu W^{-\nu}\\ &-\; \frac{\lambda v}{2}\times\sigma^3 \;-\; \frac{\lambda}{8}\times\sigma^4 \;+\; 2gv\times\sigma\times W^+_\mu W^{-\mu} \;+\; g^2\times\sigma^2\times W^+_\mu W^{-\mu}\\ &-g\times\widetilde{F}_{\mu\nu}\times\mathrm{Im}\bigl(W^{+\mu}W^{-\nu}\bigr) \;-\; g^2\times\Bigl(\mathrm{Im}\bigl(W^{+\mu}W^{-\nu}\bigr)\Bigr)^2.\end{aligned} \tag{49}$$

General Case

Let's take a closer look at eqs. (34) and (43), and focus on the mass terms for the vector fields. In both cases, we start with the kinetic terms for the original scalar fields $\Phi_i(x)$ or $\Phi^a(x)$, fix the unitary gauge, work through the algebra, and eventually obtain the kinetic term for the physical Higgs field σ, the mass terms for the vector fields — or some of the vector fields — and the interactions between the massive vectors and the Higgs σ. But is all we want are the mass terms for the vectors, we may simply freeze $\sigma(x)\equiv 0$: This would eliminate the interactions with the σ as well as the $\frac12(\partial_\mu\sigma)$ term, and all we would have left are the mass terms for the massive vectors.

Note that freezing $\sigma(x)\equiv 0$ is equivalent to freezing all the scalars at their VEVs, $\Phi(x)\equiv\langle\Phi\rangle$. Consequently, to get the vector's masses we do not need to go through the details of the

unitary gauge fixing, all we need are the scalar VEVs, then the kinetic terms for the frozen scalars

$$D_\mu\langle\Phi\rangle^\dagger\, D_\mu\langle\Phi\rangle \quad \text{or} \quad \tfrac{1}{2}(D_\mu\langle\Phi\rangle)^2$$

become the mass terms for the vectors. For example, for the $SO(3)$ triplet of real scalar fields from the second example

$$D_\mu\langle\Phi\rangle^a \;=\; -g\epsilon^{abc}A^b_\mu \times v\delta^{c3} \;=\; -gv\epsilon^{ab3}\times A^b_\mu, \tag{50}$$

$$\begin{aligned}\mathcal{L}^{\text{vector}}_{\text{mass}} \;&=\; \tfrac{1}{2}\big(D_\mu\langle\Phi\rangle^a\big)^2 \;=\; \tfrac{1}{2}(gv)^2\times\epsilon^{ab3}\epsilon^{ac3}A^b_\mu A^{c\mu}\\ &=\; \tfrac{1}{2}\big(M \;=\; gv\big)^2\times\big(A^1_\mu A^{1\mu} \,+\, A^2_\mu A^{2\mu}\big).\end{aligned} \tag{51}$$

Likewise, for the $SU(2)$ doublet of complex scalar fields from the first example,

$$D_\mu\langle\Phi\rangle^i \;=\; \frac{ig}{2}\left(A^a_\mu\sigma^a\right)^i{}_j \times \frac{v}{\sqrt{2}}\,\delta^j_2 \;=\; \frac{igv}{2\sqrt{(2)}}\times\left(A^a_\mu\sigma^a\right)^i{}_2\,, \tag{52}$$

$$D_\mu\langle\Phi\rangle^*_i \;=\; -\frac{igv}{2\sqrt{2}}\times\left(A^a_\mu\sigma^a\right)^2{}_i\,, \tag{53}$$

$$\begin{aligned}\mathcal{L}^{\text{vector}}_{\text{mass}} \;&=\; D_\mu\langle\Phi\rangle^*_i\; D^\mu\langle\Phi\rangle^i \;=\; \frac{g^2v^2}{8}\times\left(A^a_\mu\sigma^a\right)^2{}_i\left(A^{b\mu}\sigma^b\right)^i{}_2\\ &=\; \frac{g^2v^2}{8}\times A^a_\mu A^{b\mu}\times\Big[\left(\sigma^a\sigma^b\right)^2{}_2 \;=\; \delta^{ab} \,-\, i\epsilon^{ab3}\Big]\\ &=\; \frac{g^2v^2}{8}\times A^a_\mu A^{b\mu}\times\delta^{ab} \qquad \langle\!\langle\,\text{since } A^a_\mu A^{b\mu} \text{ is symmetric in } a\leftrightarrow b.\,\rangle\!\rangle\\ &=\; \frac{M^2}{2}\times A^a_\mu A^{a\mu} \qquad \text{for } M \;=\; \frac{gv}{2}\,.\end{aligned} \tag{54}$$

This recipe — freezing $\Phi(x)\equiv\langle\Phi\rangle$ to find the vector masses — applies to any kind of gauge theory with scalars in any kinds of multiplets. Indeed, consider a general gauge symmetry G with generators $\hat{T}^a$ and gauge fields $A^a_\mu(x)$ ($a = 1,\ldots,\dim(G)$). Let scalars $\Phi^\alpha(x)$ belonging to some multiplet (m) of G develop non-zero vacuum expectation values $\langle\Phi^\alpha\rangle\neq 0$. Then the covariant derivatives of these scalars

$$D_\mu\Phi^\alpha(x) \;=\; \partial_\mu\Phi^\alpha(x) \;+\; igA^a_\mu(x)\times\left(T^a_{(m)}\right)^\alpha{}_\beta\Phi^\beta(x) \tag{55}$$

become in the unitary gauge

$$D_\mu \Phi^\alpha(x) \;=\; D_\mu \langle\Phi\rangle^\alpha \;+\; \text{terms involving the physical scalars} \tag{56}$$

where

$$D_\mu \langle\Phi\rangle \;=\; igA^a_\mu(x) \times \left(T^a_{(m)}\right)^\alpha{}_\beta \langle\Phi\rangle^\beta \;. \tag{57}$$

In eq. (56), the terms involving the physical scalars — and the physical scalar fields themselves — depend on the details of the unitary gauge fixing. On the other hand, the covariant derivatives of the VEVs (57) depend only on the VEVs themselves. Moreover, such derivatives are linear functions of the vector fields with constant coefficients, so their squares become quadratic mass terms for the vectors,

$$\begin{aligned} D^\mu \langle\Phi\rangle^*_\alpha \, D_\mu \langle\Phi\rangle^\alpha \;&=\; -igA^a_\mu \times \langle\Phi\rangle^*_\beta \left(T^a_{(m)}\right)^\beta{}_\alpha \times igA^{b\mu} \times \left(T^a_{(m)}\right)^\alpha{}_\gamma \langle\Phi\rangle^\gamma \\ &=\; A^a_\mu A^{b\mu} \times g^2 \langle\Phi\rangle^*_\beta \left(T^a_{(m)} T^b_{(m)}\right)^\beta{}_\gamma \langle\Phi\rangle^\gamma \\ &\qquad \langle\!\langle \text{ by } a \leftrightarrow b \text{ symmetry of the } A^a_\mu A^{b\mu} \,\rangle\!\rangle \\ &=\; \tfrac{1}{2} A^a_\mu A^{b\mu} \times g^2 \langle\Phi\rangle^*_\beta \left\{T^a_{(m)}, T^b_{(m)}\right\}^\beta{}_\gamma \langle\Phi\rangle^\gamma \;. \end{aligned} \tag{58}$$

In other words,

$$\mathcal{L}^{\text{vector}}_{\text{masses}} \;=\; \tfrac{1}{2}\left(M^2_V\right)^{ab} \times A^a_\mu A^{b\mu} \,, \tag{59}$$

where the mass^2 matrix for the gauge fields obtains as

$$\left(M^2_V\right)^{ab} \;=\; g^2 \langle\Phi\rangle^{*\beta} \left\{T^a_{(m)}, T^b_{(m)}\right\}_\beta{}^\gamma \langle\Phi\rangle_\gamma \;\equiv\; g^2 \langle\Phi\rangle^\dagger \left\{T^a_{(m)}, T^b_{(m)}\right\} \langle\Phi\rangle \,. \tag{60}$$

To be precise, eq. (60) applies to Higgs VEVs belonging to a single multiplet of complex scalars. For a multiplet of real scalars, there is an extra factor $\frac{1}{2}$ due to different normalization of the VEVS, and for several Higgs multiplets with non-zero VEVs, the general formula is

$$\left(M^2_V\right)^{ab} \;=\; g^2 \sum_{\Phi\in(m)}^{\substack{\text{complex}\\ \text{Higgs}\\ \text{multiplets}}} \langle\Phi\rangle^\dagger \left\{T^a_{(m)}, T^b_{(m)}\right\} \langle\Phi\rangle \;+\; g^2 \sum_{\Phi\in(m)}^{\substack{\text{real}\\ \text{Higgs}\\ \text{multiplets}}} \tfrac{1}{2} \langle\Phi\rangle^\top \left\{T^a_{(m)}, T^b_{(m)}\right\} \langle\Phi\rangle \,. \tag{61}$$

In general, such mass^2 matrix is not diagonal, and we need to diagonalize in order to find the physical vector masses. For example, in the Glashow–Weinberg–Salam theory of the weak and

EM interactions — it's explained in the next set of notes — the mass matrix mixes the $SU(2)$ gauge field W^3_μ and the $U(1)$ gauge field B_μ, and the mass eigenstates are the massless EM field A_μ and the massive neural field Z_μ involved in the weak interactions.

An additional complication of the GWS theory — or any other theory with non-simple gauge group $G = G_1 \times G_2 \times \cdots$ — are different gauge couplings g for different factors G. In this case, the g^2 factor in eq. (61) for the mass^2 matrix element $(M^2)^{ab}$ should be replaced with $g(a) \times g(b)$ where $g(a)$ is the coupling of the gauge group factor containing the generator T^a, and likewise for the $g(b)$. Thus, the most general formula for the vector mass matrix stemming from the Higgs mechanism is

$$\left(M_V^2\right)^{ab} = g(a)\,g(b) \times \left[\sum_{\substack{\text{complex}\\\text{Higgs}\\\text{multiplets}}}^{\Phi\in(m)} \langle\Phi\rangle^\dagger \left\{T^a_{(m)}, T^b_{(m)}\right\} \langle\Phi\rangle \;+\; \frac{1}{2} \sum_{\substack{\text{real}\\\text{Higgs}\\\text{multiplets}}}^{\Phi\in(m)} \langle\Phi\rangle^\top \left\{T^a_{(m)}, T^b_{(m)}\right\} \langle\Phi\rangle\right]. \tag{62}$$

In my notes on the GWS theory we shall see how this works in detail, and how the gauge couplings affect the eigenstates of the mass matrix.

The Higgs scalar field with no massive Higgs particle

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The postulate that all massless elementary fields have conformal Weyl local scaling symmetry has remarkable consequences for both cosmology and elementary particle physics. Conformal symmetry couples scalar and gravitational fields. Implications for the scalar field of a conformal Higgs model are considered here. The energy-momentum tensor of a conformal Higgs scalar field determines a cosmological constant. It has recently been shown that this accounts for the observed magnitude of dark energy. The gravitational field equation forces the energy density to be finite, which precludes spontaneous destabilization of the vacuum state. Scalar field fluctuations would define a Higgs tachyon rather than a massive particle, consistent with the ongoing failure to observe such a particle.

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INTRODUCTION

The standard model of spinor and gauge boson fields has higher symmetry than does Einstein gravitational theory[1]. For massless fields with definite conformal character action integrals are invariant under local Weyl (conformal) scaling, $g_{\mu\nu}(x) \to g_{\mu\nu}(x)e^{2\alpha(x)}$[1]. A conformal energy-momentum tensor is traceless, while the Einstein tensor is not.

Compatibility can be imposed in gravitational theory by replacing the Einstein-Hilbert field action by a uniquely determined action integral I_g constructed using the conformal Weyl tensor[1]. Conformal gravity accounts for anomalous galactic rotation velocities without invoking dark matter[1]. Relativistic phenomenology at the distance scale of the solar system is preserved.

An inherent conflict between gravitational and elementary particle theory is removed if all massless elementary fields have conformal symmetry. Standard cosmology[2] postulates uniform, isotropic geometry, described by the Robertson-Walker (RW) metric tensor. In RW geometry, conformal gravitational $\mathcal{L}_g$ vanishes identically[1], but the residual gravitational effect of a conformal scalar field is consistent with Hubble expansion[1], dominated in the current epoch by dark energy, with negligible spatial curvature[3, 4].

In electroweak theory, the Higgs mechanism introduces an SU(2) doublet scalar field Φ that generates gauge boson mass[5, 6]. Postulating universal conformal symmetry for massless elementary fields, these two scalar fields can be identified[7]. Lagrangian density $\mathcal{L}_\Phi$ for conformal scalar field $\Phi(x) \to \Phi(x)e^{-\alpha(x)}$ includes a term dependent on Ricci scalar $R = g_{\mu\nu}R^{\mu\nu}$, where $R^{\mu\nu}$ is the gravitational Ricci tensor[1]. In uniform, isotropic geometry this determines a modified Friedmann cosmic evolution equation[3] consistent with cosmological data back to the microwave background epoch[4].

Implications for the standard electroweak model are examined here. The Higgs model Lagrangian density contains $\Delta\mathcal{L}_\Phi = (w^2 - \lambda\Phi^\dagger\Phi)\Phi^\dagger\Phi$, where w^2 and λ are undetermined positive constants[6]. Units here set $\hbar = c = 1$. Lagrangian term $\lambda(\Phi^\dagger\Phi)^2$ is conformally covariant. $w^2\Phi^\dagger\Phi$ breaks conformal symmetry, but can be generated dynamically[7]. Conformal symmetry requires a term $-\frac{1}{6}R\Phi^\dagger\Phi$[1]. Empirical cosmological $R > 0$[3], so $-\frac{1}{6}R$ and w^2 have opposite signs. A consistent theory must include $(w^2 - \frac{1}{6}R)\Phi^\dagger\Phi$[3].

The conformal scalar field equation has exact solutions such that $\Phi^\dagger\Phi = \phi_0^2 = (w^2 - \frac{1}{6}R)/2\lambda$, if this ratio is positive and R is treated as a constant. Only the magnitude of Φ is determined. For $\phi_0^2 > 0$, a modified Friedmann cosmic evolution equation has been derived[3] and solved to determine cosmological parameters. The residual constant term in conformal energy-momentum tensor $\Theta_\Phi^{\mu\nu}$ defines a cosmological constant (dark energy)[1, 3]. Nonzero ϕ_0^2 produces gauge boson masses[6].

Conformal theory identifies w^2 with the empirically positive cosmological constant[3], but does not specify the algebraic sign of parameter λ. For the Higgs mechanism, condition $\phi_0^2 = (w^2 - \frac{1}{6}R)/2\lambda > 0$ requires the sign of λ to agree with $w^2 - \frac{1}{6}R$. The scalar field energy density determined by the coupled equations derived here is necessarily finite for any real value of λ. This precludes destabilization of the vacuum.

Fluctuations $\delta\phi \to 0$ about an exact solution of the scalar field equation satisfy $\partial_\mu\partial^\mu\delta\phi \to -4\lambda\phi_0^2\delta\phi$. If $\lambda > 0$ this is a Klein-Gordon equation with $m_H^2 = 4\lambda\phi_0^2 = 2(w^2 - \frac{1}{6}R)$, which defines a Higgs boson[6] if $R < 6w^2$. In the conformal Higgs model, empirical values of parameters w^2, R, and ϕ_0^2 determine parameter λ. It is argued here that these parameters, now well-established from cosmological and electroweak data, imply $\lambda < 0$, consistent with an earlier formal argument[1]. Hence fluctuations of a conformal Higgs scalar field do not satisfy a Klein-Gordon equation. This rules out a standard Higgs particle of any real mass. Negative m_H^2, or finite pure imaginary mass, would define a tachyon[8], if such a particle or field could exist, and might justify an experimental search for such a tachyon.

THE MODIFIED FRIEDMANN EQUATION

In cosmological theory, a uniform, isotropic universe is described by Robertson-Walker (RW) metric $ds^2 = dt^2 - a^2(t)(\frac{dr^2}{1-kr^2} + r^2 d\omega^2)$, if $c = \hbar = 1$ and $d\omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. Gravitational field equations are determined by Ricci tensor $R^{\mu\nu}$ and scalar R. The RW metric defines two independent functions $\xi_0(t) = \frac{\ddot{a}}{a}$ and $\xi_1(t) = \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}$, such that $R^{00} = 3\xi_0$ and $R = 6(\xi_0 + \xi_1)$. The field equations reduce to Friedmann equations for scale factor $a(t)$ and Hubble function $h(t) = \frac{\dot{a}}{a}(t)$.

If the scalar field required by Higgs symmetry-breaking has conformal symmetry, its action integral I_Φ must depend on the Ricci scalar, implying a gravitational effect. Because conformal gravitational action integral I_g vanishes identically in RW geometry[1], it is consistent to assume that uniform cosmological gravity is determined by this scalar field.

Including term $(w^2 - \frac{1}{6}R)\Phi^\dagger\Phi$ in $\mathcal{L}_\Phi$[3], the field equation for scalar Φ is $\partial_\mu\partial^\mu\Phi = (w^2 - \frac{1}{6}R - 2\lambda\Phi^\dagger\Phi)\Phi$. Generalizing the Higgs construction, and neglecting the cosmological time derivative of R, constant $\Phi = \phi_0$ is a global solution if $\phi_0^2 = \frac{1}{2\lambda}(w^2 - \frac{1}{6}R)$. Evaluated for this field solution, $\mathcal{L}_\Phi = \phi_0^2(w^2 - \frac{1}{6}R - \lambda\phi_0^2) = \frac{1}{2}\phi_0^2(w^2 - \frac{1}{6}R)$.

Variational formalism of classical field theory[9] is easily extended to the context of general relativity[1]. The metric functional derivative $\frac{1}{\sqrt{-g}}\frac{\delta I}{\delta g_{\mu\nu}}$ of generic action integral $I = \int d^4x\sqrt{-g}\mathcal{L}$ is $X^{\mu\nu} = x^{\mu\nu} + \frac{1}{2}g^{\mu\nu}\mathcal{L}$, if $\delta\mathcal{L} = x^{\mu\nu}\delta g_{\mu\nu}$. The energy-momentum tensor is $\Theta^{\mu\nu} = -2X^{\mu\nu}$. Varying $g_{\mu\nu}$ for fixed scalar field solution Φ, metric functional derivative

$$X_\Phi^{\mu\nu} = \frac{1}{6}R^{\mu\nu}\Phi^\dagger\Phi + \frac{1}{2}g^{\mu\nu}\mathcal{L}_\Phi$$
$$= \frac{1}{6}\phi_0^2(R^{\mu\nu} - \frac{1}{4}Rg^{\mu\nu} + \frac{3}{2}w^2 g^{\mu\nu}) \quad (1)$$

implies modified Einstein and Friedmann equations[3].

The gravitational field equation driven by energy-momentum tensor $\Theta_m^{\mu\nu} = -2X_m^{\mu\nu}$ for uniform matter and radiation is $X_\Phi^{\mu\nu} = \frac{1}{2}\Theta_m^{\mu\nu}$. Since $\Theta_m^{\mu\nu}$ is finite, determined by fields independent of Φ, $X_\Phi^{\mu\nu}$ must be finite, regardless of any parameters of the theory. This precludes spontaneous destabilization of the conformal Higgs model.

Defining $\bar\kappa = -3/\phi_0^2$ and $\bar\Lambda = \frac{3}{2}w^2$, the modified Einstein equation is

$$R^{\mu\nu} - \frac{1}{4}Rg^{\mu\nu} + \bar\Lambda g^{\mu\nu} = -\bar\kappa\Theta_m^{\mu\nu}. \quad (2)$$

Traceless conformal tensor $R^{\mu\nu} - \frac{1}{4}Rg^{\mu\nu}$ here replaces the Einstein tensor of standard theory[3]. Cosmological constant $\bar\Lambda$ is determined by Higgs parameter w^2. Nonstandard parameter $\bar\kappa < 0$ is determined by the scalar field[1, 3]. For energy density $\rho = \Theta_m^{00}$ this implies $-\frac{2}{3}(R^{00} - \frac{1}{4}R) = \xi_1(t) - \xi_0(t) = \frac{2}{3}(\bar\kappa\rho + \bar\Lambda)$. Hence uniform, isotropic matter and radiation determine the modified Friedmann cosmic evolution equation[3]

$$\xi_1(t) - \xi_0(t) = \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \frac{\ddot{a}}{a} = \frac{2}{3}(\bar\kappa\rho + \bar\Lambda). \quad (3)$$

Because the trace of $R^{\mu\nu} - \frac{1}{4}Rg^{\mu\nu}$ is identically zero, a consistent theory must satisfy the trace condition $g_{\mu\nu}\bar\Lambda g^{\mu\nu} = 4\bar\Lambda = -\bar\kappa g_{\mu\nu}\Theta_m^{\mu\nu}$. From the definition of an energy-momentum tensor, this is just the trace condition satisfied in conformal theory[10], $g_{\mu\nu}(X_\Phi^{\mu\nu} + X_m^{\mu\nu}) = 0$. Vanishing trace eliminates the second Friedmann equation derived in standard theory. Although the w^2 term in $\Delta\mathcal{L}_\Phi$ breaks conformal symmetry, a detailed argument shows that the trace condition is preserved[7].

FITS TO COSMOLOGICAL DATA

The modified Friedmann equation determines dimensionless scale parameter $a(t) = 1/(1 + z(t))$, for redshift $z(t)$, and function $h(t) = \frac{\dot{a}}{a}(t)$ in units of current Hubble constant H_0 =70.5 km/s/Mpc[4], such that $z = 0, a = 1, h = 1$ at present time t_0. Distances here are in Hubble units c/H_0.

The modified Friedmann equation depends on nominally constant parameters, fitted to cosmological data for $z \le z_*$: $\alpha = \frac{2}{3}\bar\Lambda = w^2 > 0$, $k \simeq 0$, $\beta = -\frac{2}{3}\bar\kappa\rho_m a^3 > 0$, and $\gamma = 3\beta/4R_b(t_0)$. $z_* = 1090$ here characterizes the cosmic microwave background, at t_*, when radiation became decoupled from matter. $\frac{4}{3}R_b(t)$ is the ratio of baryon to radiation energy densities. Empirical value $R_b(t_0) = 688.6$[4] is assumed. Scaled energy densities $\rho_m a^3$ and $\rho_r a^4$, for matter and radiation respectively, are constant. In the absence of dark matter, $\rho_m \simeq \rho_b$, the baryon density.

The parametrized modified Friedmann equation is

$$\frac{\dot{a}^2}{a^2} - \frac{\ddot{a}}{a} = -\frac{d}{dt}\frac{\dot{a}}{a} = \hat\alpha = \alpha - \frac{k}{a^2} - \frac{\beta}{a^3} - \frac{\gamma}{a^4}. \quad (4)$$

Dividing this equation by $h^2(t)$ implies dimensionless sum rule

$$\Omega_m(t) + \Omega_r(t) + \Omega_\Lambda(t) + \Omega_k(t) + \Omega_q(t) = 1, \quad (5)$$

where $\Omega_m(t) = \frac{2}{3}\frac{\bar\kappa\rho_m(t)}{h^2(t)} < 0$, $\Omega_r(t) = \frac{2}{3}\frac{\bar\kappa\rho_r(t)}{h^2(t)} < 0$, $\Omega_\Lambda(t) = \frac{w^2}{h^2(t)} > 0$, $\Omega_k(t) = -\frac{k}{a^2(t)h^2(t)}$, and $\Omega_q(t) = \frac{\ddot{a}a}{\dot{a}^2} = -q(t)$. In contrast to the standard sum rule, Ω_m and Ω_r are negative, while acceleration parameter $\Omega_q(t)$ appears explicitly.

Hubble expansion is characterized for type Ia supernovae by scaled luminosity distance d_L as a function of redshift z. Here $d_L(z) = (1 + z)d_z$, for geodesic distance d_z corresponding to $r_z = \int c dt/a(t)$, integrated from t_z to t_0. In curved space (for $k < 0$), $d_z = \frac{\sinh(\sqrt{-k}r_z)}{\sqrt{-k}}$. In the

standard ΛCDM model[2], radiation density and curvature Ω_k can be neglected in the current epoch ($z \leq 1$). This reduces the sum rule to $\Omega_\Lambda + \Omega_m = 1$. Empirical value $\Omega_\Lambda = 0.726$ forces Ω_m to be much larger than can be accounted for by observed matter, providing a strong argument for dark matter. Mannheim[1, 11] questioned this implication, and showed that observed luminosities could be fitted equally well for $z \leq 1$ with $\Omega_m = 0$, using the standard Friedmann equation. However, sum rule $\Omega_\Lambda + \Omega_k = 1$ then requires an empirically improbable large curvature parameter Ω_k. Empirical limits are $\Omega_k \simeq \pm 0.01$[4].

This issue was examined by solving the modified Friedmann equation with parameters k, β, γ set to zero[3]. Ω_q is determined by the solution. The modified sum rule $\Omega_\Lambda + \Omega_q = 1$ then presents no problem. Computed $d_L(z)$ agrees with Mannheim's empirical function for $z \leq 1$ to graphical accuracy, using parameter $\alpha = \Omega_\Lambda(t_0) = 0.732$ for $\Omega_k(t_0) = 0$. This is consistent with current empirical values $\Omega_\Lambda = 0.726 \pm 0.015, \Omega_k = -0.005 \pm 0.013$[4]. Ω_m and Ω_r can apparently be neglected for $z \leq 1$.

$t = 0$ is defined by $h(t) = 0$ in the conformal model, which describes an initial inflationary epoch[3]. The modified Friedmann equation was solved numerically for $0 \leq t \leq t_0$[3], with parameters fitted to $d_L(z)$ for $z \leq 1$, to shift parameter $R(z_*)$[12], and to acoustic scale ratio $\ell_A(z_*)$[12]. This determines model parameters $\alpha = 0.7171, k = -0.01249, \beta = 0.3650 \times 10^{-5}$. Fixed at $\gamma = 3\beta/4R_b(t_0)$, which neglects dark matter, parameter $\gamma = 0.3976 \times 10^{-8}$. There is no significant inconsistency with model-independent empirical data[4].

Defining $\zeta = \frac{1}{6}R - w^2$, the dimensionless sum rule determines $\zeta = \xi_0 + \xi_1 - w^2 = h(t)^2(2\Omega_q + \Omega_m + \Omega_r)$. For $a \to 0$, when both α and k can be neglected, the sum rule implies $\zeta = h(t)^2(\Omega_q + 1)$. For large a, $\zeta = h(t)^2(2\Omega_q)$. $\zeta > 0$ in both limits, regardless of numerical values, since $\Omega_q > 0$. The present empirical parameters imply that ζ is positive for all z[3].

Conformal symmetry is consistent with any real value of parameter λ. However, in electroweak theory Higgs symmetry-breaking requires nonvanishing conformal scalar field Φ[5]. A positive value of ζ implies

$$\lambda\phi_0^2 = \frac{1}{2}(w^2 - \frac{1}{6}R) = -\frac{1}{2}\zeta < 0. \quad (6)$$

As argued above, for $\phi_0^2 > 0$ this conflicts with existence of the hypothetical massive Higgs boson.

DYNAMICAL ESTIMATE OF PARAMETER w^2

Since term $w^2\Phi^\dagger\Phi$ in standard parametrized $\Delta\mathcal{L}$ breaks conformal symmetry, it must be generated dynamically in a consistent theory[10]. As shown above, this term accounts for dark energy. Dynamically induced w^2 preserves the conformal trace condition[7].

The Higgs model deduces gauge boson mass from an exact solution of the parametrized scalar field equation[6]. For interacting fields, this logic can be extended to deduce nominally constant field parameters from a solution of the coupled field equations. Such a solution of nonlinear equations does not depend on linearization or on perturbation theory.

Interaction of scalar and gauge boson fields defines a quasiparticle scalar field in Landau's sense: Φ is dressed via virtual excitation of accompanying gauge fields. The derivation summarized here considers gravitational field $g_{\mu\nu}$ interacting with scalar field Φ and $U(1)$ gauge field B_μ. Solution of the coupled semiclassical field equations[7] gives an order-of-magnitude estimate of parameter w^2, in agreement with the empirical cosmological constant, while confirming the Higgs formula for gauge boson mass[5, 6].

The conformal Higgs model assumes incremental Lagrangian density $\Delta\mathcal{L}_\Phi = w^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2$, with undetermined numerical parameters w^2 and λ. The implied scalar field equation is $\partial_\mu\partial^\mu\Phi + \frac{1}{6}R\Phi = \frac{1}{\sqrt{-g}}\frac{\delta\Delta I}{\delta\Phi^\dagger} = (w^2 - 2\lambda\Phi^\dagger\Phi)\Phi$. If R, w^2, λ are constant, this has an exact solution $\Phi^\dagger\Phi = \phi_0^2 = (w^2 - \frac{1}{6}R)/2\lambda$, if this ratio is positive. For massive complex vector field B_μ, parametrized $\Delta\mathcal{L}_B$ implies field equation $\partial_\nu B^{\mu\nu} = 2\frac{1}{\sqrt{-g}}\frac{\delta\Delta I}{\delta B^*_\mu} = m_B^2 B^\mu - J_B^\mu$.

For interacting fields, both $\Delta\mathcal{L}_\Phi$ and $\Delta\mathcal{L}_B$ can be identified with incremental Lagrangian density $\Delta\mathcal{L} =$

$$\frac{i}{2}g_b B^\mu(\partial_\mu\Phi)^\dagger\Phi - \frac{i}{2}g_b B^\dagger_\mu\Phi^\dagger\partial^\mu\Phi + \frac{1}{4}g_b^2\Phi^\dagger B^\dagger_\mu B^\mu\Phi, \quad (7)$$

due to covariant derivatives, with coupling constant g_b. Evaluated for solutions of the coupled field equations,

$$2\frac{1}{\sqrt{-g}}\frac{\delta\Delta I}{\delta B^*_\mu} = \frac{1}{2}g_b^2\Phi^\dagger\Phi B^\mu - ig_b\Phi^\dagger\partial^\mu\Phi \quad (8)$$

implies Higgs mass formula $m_B^2 = \frac{1}{2}g_b^2\phi_0^2$. The fields are coupled by current density $J_B^\mu = ig_b\Phi^\dagger\partial^\mu\Phi$. For the scalar field, neglecting derivatives of B_μ,

$$\frac{1}{\sqrt{-g}}\frac{\delta\Delta I}{\delta\Phi^\dagger} = \frac{1}{4}g_b^2 B^*_\mu B^\mu\Phi - \frac{i}{2}g_b(B^*_\mu + B_\mu)\partial^\mu\Phi \quad (9)$$

implies $w^2 = \frac{1}{4}g_b^2 B^*_\mu B^\mu$.

For $\zeta = \frac{1}{6}R - w^2 > 0$, $\Phi^\dagger\Phi = \phi_0^2 = -\zeta/2\lambda$ solves the scalar field equation if $\lambda < 0$. Ricci scalar $R(t)$ varies on a cosmological time scale, so that $\frac{\dot\phi_0}{\phi_0} = \frac{1}{2}\frac{\dot R}{R - 6w^2} \neq 0$, for constant w^2 and λ. This implies small but nonvanishing real $\frac{\dot\phi_0}{\phi_0}$, hence nonzero pure imaginary source current density $J_B^0 = ig_b\phi_0^*\partial^0\phi_0 = ig_b\frac{\dot\phi_0}{\phi_0}\phi_0^2$.

Derivatives due to cosmological time dependence act as a weak perturbation of SU(2) scalar field solution $\Phi = (\Phi_+, \Phi_0) \to (0, \phi_0)$. Neglecting extremely small derivatives of the induced gauge fields (but not of Φ), the gauge field equation reduces to $m_B^2 B^\mu = J_B^\mu$. Implied pure imaginary B^μ does not affect parameter λ.

The coupled field equations imply $w_B^2 = \frac{1}{4}g_b^2|B|^2$, proportional to $(\frac{\dot\phi_0}{\phi_0})^2$. Since observable properties depend only on $|B|^2$, a pure imaginary virtual field implies no obvious physical inconsistency. Gauge symmetry is broken in any case by a fixed field solution. The scalar field is dressed by the induced gauge field.

Numerical solution of the modified Friedmann equation[3, 7] implies $\zeta(t_0) = 1.224 \times 10^{-66} eV^2$, at present time t_0. Given $\phi_0 = 180 GeV$[6], $\lambda = -\frac{1}{2}\zeta/\phi_0^2 = -0.189 \times 10^{-88}$.

U(1) gauge field B_μ does not affect λ. Using $|B|^2 = |J_B|^2/m_B^4, |J_B|^2 = g_b^2(\frac{\dot\phi_0}{\phi_0})^2\phi_0^4$ and $m_B^2 = \frac{1}{2}g_b^2\phi_0^2$, the dynamical value of w^2 due to B_μ is $w_B^2 = \frac{1}{4}g_b^2|B|^2 = (\frac{\dot\phi_0}{\phi_0})^2$.

From the solution of the modified Friedmann equation[7], $\frac{\dot\phi_0}{\phi_0}(t_0) = -2.651$ and $w_B^2 = 7.027$, in Hubble units, so that $w_B = 2.651\hbar H_0 = 3.984 \times 10^{-33} eV$ in energy units. This can be considered only an order-of-magnitude estimate, since time dependence of the assumed constants, implied by the present theory, was not considered in fitting empirical cosmological data[3]. Moreover, the SU(2) gauge field has been omitted.

NOTE ON DARK MATTER

As stated in[3], interpretation of parameter Ω_m may require substantial revision of the standard cosmological model. Directly observed inadequacy of Newton-Einstein gravitation may imply the need for a modified theory rather than for inherently unobservable dark matter.

Mannheim has applied conformal gravity to anomalous galactic rotation[1], fitting observed data for a set of galaxies covering a large range of structure and luminosity. The role played in standard ΛCDM by dark matter, separately parametrized for each galaxy, is taken over in conformal theory for Schwarzschild geometry by an external linear radial potential. The remarkable fit to observed data shown in[1][Sect.6.1,Fig.1] requires only two universal parameters for the whole set of galaxies.

As discussed by Mannheim[1][Sects.6.3,9.3], a significant conformal contribution to centripetal acceleration is independent of total galactic luminous mass. This implies an external cosmological source. Such an isotropic source would determine an inherently spherical halo of gravitational field surrounding any galaxy. Quantitative results for lensing and for galactic clusters should be worked out before assuming dark matter.

CONCLUSIONS

This paper is concerned with determining parameters w^2 and λ in the incremental Lagrangian density of the Higgs model, $\Delta\mathcal{L}_\Phi = (w^2 - \lambda\Phi^\dagger\Phi)\Phi^\dagger\Phi$. Fitting the modified Friedmann equation to cosmological data[3] implies dark energy parameter $\Omega_\Lambda = w^2 = 0.717$, so that empirical $w = \sqrt{0.717}\hbar H_0 = 1.273 \times 10^{-33} eV$.

The modified Friedmann equation determines the time derivative of the cosmological Ricci scalar, which implies nonvanishing source current density for induced U(1) gauge field B_μ, treated here as a classical field in semiclassical coupled field equations. The resulting gauge field intensity estimates the U(1) contribution to w^2 such that $w_B = 2.651\hbar H_0 = 3.984 \times 10^{-33} eV$. This order-of-magnitude agreement between computed w_B and empirical w supports the conclusion that conformal theory explains both the existence and magnitude of dark energy[7].

The present argument obtains an accurate empirical value of parameter λ from the known dark energy parameter[4], from the implied current value of Ricci scalar R[3], and from scalar field amplitude ϕ_0 determined by gauge boson masses[6]. The mass parameter for a fluctuation of the conformal Higgs scalar field satisfies $m_H^2 = 4\lambda\phi_0^2$. Empirical value $\lambda = -0.189 \times 10^{-88}$ is negative, implying finite pure imaginary parameter m_H. If such a particle or field could exist or be detected, this would define a tachyon[8], the quantum version of a classical particle that moves more rapidly than light. Experimental data rule out a standard massive Higgs boson with mass $0 \leq m_H \leq 108$GeV[13, 14]. However, a Higgs tachyon[8] might either not exist at all, or elude detection in experiments designed for a classical massive Higgs boson. The present results would only be inconsistent if experimental Higgs searches to date were capable of detecting a Higgs tachyon and failed to do so. Conformal theory clearly rules out a standard Higgs boson in the multi-GeV range.

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BOSONS SCALAIRES et SUPERSYMÉTRIE

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Institut Henri Poincaré, Paris, 29 novembre 2014

Après les deux exposés précédents

François Englert: " Le boson de Brout-Englert-Higgs "

Yves Sirois : " La découverte du boson ***H*** au LHC "

On sait beaucoup de choses sur les bosons scalaires

mais pas tout encore ...

On peut notamment se demander:

est-ce bien le **boson scalaire du Modèle Standard ?**

y en a-t-il d'autres $\left\{ \begin{array}{l} \text{de la même sorte } \textbf{?} \\ \text{ou d'une autre sorte } \textbf{?} \end{array} \right.$

et *quelle* ***interprétation plus profonde*** *pourrait-on aussi donner*

à celui qu'on vient de trouver **?**

Quelles réponses ?

Le boson trouvé à 125 GeV présente bien, à ce stade,

les caractéristiques attendues du boson de Brout-Englert-Higgs du Modèle Standard

*   *
*

Mais bien d'autres pourraient aussi exister ...

notamment dans le cadre de la supersymétrie

*   *
*

et enfin ...

Le BOSON BEH est-il ... UN *Z* SANS SPIN ?

il faut déjà savoir ce qu'est

un **Boson de Brout-Englert-Higgs** *spin 0*

particule associée, notamment, à ***l'origine des masses***

et à la brisure de la symétrie électrofaible

et un Z ... *spin 1*

médiateur neutre de l'interaction faible

responsable des diffusions de neutrinos par la matière, $\nu + p \to \nu + X$ ...

observées au CERN en 1973

Z se couple au "***courant neutre faible***"

(comme le photon au courant électromagnétique)

découverte du Z en 1983 (masse 91 GeV$/c^2$)

DEUX SORTES DE PARTICULES ...

$$\begin{cases} \textit{Bosons}: & \textit{particules de spin entier, 0, 1, 2, ..., en unité } \hbar \\ \textit{Fermions}: & \textit{particules de spin demi-entier, } \frac{1}{2},\ \frac{3}{2},\ \dots,\ \textit{en unité } \hbar \end{cases}$$

Pour les particules fondamentales :

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médiateurs des interactions

Bosons de spin 1 : $\boxed{W^+,\ W^-,\ Z,\ \textbf{photon},\ \textbf{gluons}}$

(pas de boson de spin 0 pour le moment ...)

"constituants de la matière" (au sens large)

électron, ... , neutrinos, quarks ($\rightarrow$ *proton* = uud, *neutron* = ddu)

Fermions de spin $\frac{1}{2}$:

$$\boxed{\begin{matrix} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} & \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} & \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \\ \begin{pmatrix} u \\ d \end{pmatrix} & \begin{pmatrix} c \\ s \end{pmatrix} & \begin{pmatrix} t \\ b \end{pmatrix} \end{matrix}}$$

Les BOSONS ...

(spin entier, 0, 1, 2, ... , en unité $\hbar$)

spin 1 $\left\{ \begin{array}{lll} \boldsymbol{Z}: & \underline{\textbf{médiateur neutre de l'interaction faible}} & m_Z \simeq \underline{\underline{\mathbf{91}}}\ \mathrm{GeV}/c^2 \\ W^+,\ W^-: & \textit{médiateurs chargés de l'interaction faible} & m_W \simeq 80\ \mathrm{GeV}/c^2 \\ \text{photon } (\gamma): & \textit{médiateur (neutre) de l'int. électromagnétique} & m_\gamma = 0 \end{array} \right.$

le mécanisme de Brout-Englert-Higgs *(1964)*

permet de donner des masses aux $W^\pm$ *et* Z *(Weinberg, 1967)*

Un électron dans un champ électromagnétique acquiert une énergie électrostatique $E = qV$

Il peut aussi **interagir avec** **un champ de spin 0** ϕ

et acquérir une masse $\boldsymbol{m_e = \lambda_e\, \phi}$

(le champ ϕ *étant ici supposé uniforme dans tout l'espace)*

ondes $\rightarrow$ **quanta** *associés à ce champ* =

BOSONS "de Brout-Englert-Higgs" **de spin 0**

Plus précisément: $\boxed{\textit{LA THÉORIE ÉLECTROFAIBLE}}$ (1967)

φ doublet de champs de spin 0 complexe $\Leftrightarrow$ 4 composantes réelles (3 phases et 1 module)

Trois composantes éliminées par le **mécanisme BEH**, pour donner des masses aux $\boldsymbol{W^\pm}$ et $\boldsymbol{Z}$

Reste la quatrième, $\boxed{\boldsymbol{\phi = \sqrt{2\,\varphi^\dagger\varphi}}}$ qui s'ajuste pour minimiser $\boldsymbol{V(\varphi) = \lambda\,(\varphi^\dagger\varphi)^2 - \mu^2\,\varphi^\dagger\varphi}$

$$\phi = v = \sqrt{\mu^2/\lambda}$$

quanta associés au champ $\phi \to$ bosons BEH, de spin 0

La symétrie de jauge reste exacte (et même non brisée) mais maintenant cachée

(elle est dite ordinairement "spontanément brisée")

Différenciation entre interactions faibles (courte portée) ***et électromagnétique*** (longue portée) :

$$\boldsymbol{m_W} = \frac{gv}{2}\,, \quad \boldsymbol{m_Z} = \frac{\sqrt{g^2+g'^2}\,v}{2} = m_W/\cos\theta\,; \quad m_\gamma = 0\,; \quad \tan\theta = g'/g, \quad e = g\,\sin\theta$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} = \frac{1}{2v^2} \quad \Longrightarrow \quad \boldsymbol{v} = (G_F\sqrt{2})^{-1/2} \simeq 246 \text{ GeV.}$$

Couplages scalaires aux quarks et aux leptons, proportionnels aux masses:

$$\boldsymbol{\lambda_{q,l} = \frac{m_{q,l}}{v} = 2^{1/4}\,G_F^{1/2}\,m_{q,l}}$$

Mais quelle est la masse du boson BEH ??

comme chacun sait, $m_H = \mu\sqrt{2} = \sqrt{2\lambda v^2}$ ....

mais que vaut λ ??

$$\boxed{\textit{40 ans plus tôt, en SUPERSYMÉTRIE, déjà ...}}$$

PF, NPB 90, 104 (1975)

il faut 2 doublets de champs de spin 0 pour la brisure électrofaible (+ éventuel singulet)

$$h_1 = \begin{pmatrix} h_1^0 \\ h_1^- \end{pmatrix} \quad \text{et} \quad h_2 = \begin{pmatrix} h_2^+ \\ h_2^0 \end{pmatrix}$$

$\Rightarrow$ *nouveaux bosons BEH, chargés et neutres* $H^\pm;\ H,\ h,\ A\ldots$

couplages quartiques fixés par g^2 et g'^2, en particulier $(g^2+g'^2)/8$

$\Rightarrow$ ***boson neutre de spin 0, de même masse que le Z***

$$\boxed{m_h = \sqrt{2\lambda\, v^2} = \frac{\sqrt{g^2+g'^2}\, v}{2} = m_Z \simeq 91\ \mathrm{GeV}/c^2}$$

tant que les effets de brisure de supersymétrie ne se font pas sentir

La supersymétrie fournit un cadre naturel pour un boson scalaire de masse

$$\boxed{m_h = m_Z \simeq 91 \text{ GeV}/c^2}$$

avant effets de brisure de supersymétrie

un boson BEH, qui est aussi $\boxed{\textit{un } Z \textit{ de spin 0}}$

* *
*

Le boson scalaire du modèle standard est longtemps resté

sa dernière pièce manquante

après la découverte du quark top en 1995

échappant jusqu'en 2012 à tous les recherches expérimentales

notamment au LEP, qui a établi une borne inférieure de 114 GeV/c^2 sur sa masse

* *
*

Beaucoup ont longtemps mis en doute son existence réelle

particulièrement à partir de la fin des années 1970

mais pourquoi ?

Un boson scalaire, élémentaire ou pas ?

l'existence d'un tel scalaire a été beaucoup mise en question

de nombreux physiciens ayant longtemps eu

des doutes sérieux sur l'existence même de champs de spin 0 fondamentaux

En présence de grandes échelles d'énergie ($\gg$ électrofaible)

telles $m_{GUT} \approx 10^{16}$ GeV ou $m_{\text{Planck}} \simeq 10^{19}$ GeV

ceux-ci devraient tendre à acquérir de grandes masses,

et disparaître de la théorie de basse énergie

Nombreux efforts visant à remplacer les champs de spin 0 fondamentaux

par des champs composés de champs de spin 1/2

e.g. des champs de ***techniquarks*** *spécialement introduits dans ce but*

*avec une nouvelle interaction de "****technicouleur****" ($SU(4)_{TC}$), puis de "technicouleur étendue"*

dans l'espoir d'éviter les champs scalaires fondamentaux

(sans grand succès à ce jour)

Un NOUVEAU BOSON ...

Comme chacun sait

Le LHC du CERN a découvert en 2012 une nouvelle particule

depuis longtemps activement recherchée – car nécessaire à la cohérence de la théorie

de masse $\simeq$ **125 GeV**$/c^2$

Elle se désintègre en $\gamma\gamma$, WW^*, ZZ^*, $b\bar{b}$, $\tau^+\tau^-$, ...

C'est un BOSON, presque sûrement **de spin 0** *(plutôt que 2)*

que l'on pense être le (ou un)

Boson de Brout-Englert-Higgs

associé à la **brisure spontanée de la symétrie électrofaible** $SU(2) \times U(1)$

et à l'origine des masses $(m_W, m_Z, m_e, ...)$

Le boson de Brout-Englert-Higgs est la dernière pièce manquante du

MODÈLE STANDARD *de la physique des particules*

après les découvertes

des **courants faibles neutres** (1973)

du **quark charmé** c (1974-76)

des **gluons** (1979)

des bosons intermédiaires $W^{\pm}$ et Z (1983)

et du **quark top** t (1995)

* * *

Obtient-on alors, avec un **boson** H *scalaire* (2012)

et un Modèle Standard qui serait finalement complet,

une description satisfaisante de la physique des particules ?

LE MODÈLE STANDARD

interactions fortes, électromagnétiques et faibles des quarks et leptons

$$SU(3) \times SU(2) \times U(1)$$

bosons de jauge (spin 1) :	**gluons,** $\underbrace{W^+,\ W^-,\ Z,\ \text{photon}}$
fermions (spin-$\frac{1}{2}$) :	**quarks, leptons**

+ 1 boson de Brout-Englert-Higgs de spin 0

associé à la **brisure spontanée de la symétrie électrofaible** *et à* **l'origine des masses**

potentiel en "chapeau mexicain" ...

– *succès remarquables*

– *mais laisse de nombreuses questions sans réponse*

quelques questions

Est-ce bien le boson scalaire du Modèle Standard ?

(et aurait-on alors tout compris ?)

ou cette particule pourrait-elle avoir des propriétés légèrement différentes,

signes d'une **"nouvelle physique"** *au delà du Modèle Standard ?*

Pas de tel signe pour l'instant. Les études se poursuivent ...

* * *

Est-il le seul ?

ou peut-il en exister d'autres – neutres ou chargés ?

comme dans les théories supersymétriques:

au moins ***5 bosons tels bosons scalaires***, *deux chargés et 3 neutres:*

$$\textit{SUSY} \quad \Rightarrow \quad \boxed{H^+,\ H^-;\ H,\ h,\ A,\ \ \ldots}$$

Comment mieux ***comprendre et interpréter ce boson H ?***

Un seul champ de spin-0 ? ou plusieurs (comme en supersymétrie) **?**

est-il fondamental, ou composé *(mais alors de quoi ...)* ?

$* \quad * \quad *$

articulation avec les champs de spin 1 ou $\frac{1}{2}$?

d'où provient son potentiel – et donc sa masse ?

comment l'échelle de masse associée (≈ 100 GeV) peut-elle rester modérée ?

$* \quad * \quad *$

Quoi qu'il en soit,

le Modèle Standard ne peut être la fin de l'histoire, il doit exister de la

Nouvelle Physique au delà du Modèle Standard

(*mais laquelle ... ?*)

NOMBREUSES QUESTIONS, dont

pourquoi **3 familles de quarks et de leptons**

qu'est-ce qui détermine leurs masses, et angles de mélange

d'où proviennent les (très petites) masses des neutrinos ?

$*\quad *$
$*$

problèmes liés à la conservation ou non-conservation des symétries $\boldsymbol{P}$ et $\boldsymbol{CP}$ ...

origine de la **prépondérance de la matière sur l'antimatière**

$*\quad *$
$*$

nature de la **matière sombre** (non-baryonique) de l'Univers ($\simeq\ 26\,\%$)

et de l' **énergie sombre** qui semble responsable de l'accélération de son expansion ($\simeq\ 69\,\%$)

provient-elle d'une **constante cosmologique $\boldsymbol{\Lambda}$** ?

mais pourquoi celle-ci serait-elle aussi incroyablement petite ? ($\Lambda <\ 10^{-121}\ L_{\text{Planck}}^{-2}$)

$*\quad *$
$*$

*la **gravitation**, et son inclusion dans le cadre quantique ...*

$\rightarrow$ **rôle de l'ESPACE-TEMPS**

et de sa généralisation à des coordonnées supplémentaires

("ordinaires": x^5, x^6 ; ou anticommutantes: θ^α, comme en supersymétrie)

$$\longrightarrow$$

Dimensions supplémentaires

$$x^{\hat\mu} = (x^\mu,\ x^5,\ x^6 ...)$$

dim. spatiales, très petites

(x^5, x^6 typiquement $< 10^{-16}$ **cm**)

Superespace

$$(x^\mu,\ \theta)$$

dim. fermioniques, et anticommutantes

$$\theta^\alpha\theta^\beta = -\,\theta^\beta\theta^\alpha$$

$\Rightarrow$ **Supersymétrie**

(les deux approches peuvent être combinées)

Autres questions ...

l'éventuelle unification des interactions

La nature de la MATIÈRE SOMBRE (non-baryonique) de l'Univers

? ?

des particules neutres, suffisamment massives,

ayant survécu aux annihilations des premiers instants de l'Univers ... (?)

(mais pas dans le modèle standard ...)

alors, une nouvelle sorte de particules ?

pourquoi seraient-elles stables ?

il serait bien d'avoir une raison fondamentale ...

$\rightarrow$

... la supersymétrie $\rightarrow$ *le* ***neutralino***
... la ***R-parité*** $\rightarrow$ *sa stabilité*

nouvelles particules, et symétries ...

LA SUPERSYMÉTRIE

peut *(en principe)* **relier les bosons et les fermions**

en changeant le spin des particules d'une demi unité

Transformations géométriques dans le superespace

L'espace-temps $x^\mu = \begin{pmatrix} ct \\ \vec{x} \end{pmatrix}$ *est étendu au* ***superespace*** $(\boldsymbol{x^\mu},\, \boldsymbol{\theta})$

$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{pmatrix}$ *coordonnées anticommutantes (de spin* $\frac{1}{2}$ *):* $\theta_i \theta_j \;=\; -\,\theta_j \theta_i \;, \quad (\theta_i)^2 \;=\; 0$

$$\uparrow$$

(cf. Principe de Pauli: deux fermions identiques ne peuvent être dans le même état quantique)

Algèbre : $\left\{ \begin{array}{rcl} \{\, \boldsymbol{Q} \,,\, \bar{\boldsymbol{Q}} \,\} & = & -\,\mathbf{2}\, \gamma_\mu\, \boldsymbol{P}^\mu \\ [\, \boldsymbol{Q},\ \boldsymbol{P}^\mu\,] & = & \mathbf{0} \end{array} \right.$

engendre les translations ...

$$\boxed{\textit{\textbf{BOSONS}} \quad \overset{\textit{SUPERSYMÉTRIE}}{\longleftrightarrow} \quad \textit{\textbf{FERMIONS}}}$$

c'est aussi le cadre naturel pour discuter de particules de spin 0 fondamentales ...

(les mal-aimées de la théorie des champs)

peut-on alors relier les Bosons, **messagers des interactions**

aux Fermions **constituants de la matière** ?

et arriver à une sorte d'unification

$$\boxed{\textbf{FORCES} \;\leftrightarrow\; \textbf{MATIÈRE} \;??}$$

idée très attirante !

mais les choses ne se passent pas ainsi ... !!

(contrairement à ce qui est souvent écrit ...)

Comment la Nature pourrait-elle être supersymétrique ???

il ne semble pas que ce puisse être le cas ...

Comment briser spontanément la supersymétrie ?

alors qu'un état supersymétrique a toujours moins d'énergie qu'un autre qui ne l'est pas ...

et si l'on y arrive, pourquoi le fermion de Goldstone associé n'est-il pas observé ?

Que faire des fermions de Majorana de ces théories ?

alors que les quarks et les leptons sont de Dirac ?

Comment définir des nombres quantiques "fermioniques" B et L conservés

comment éviter des échanges de particules de spin 0, rendant le proton très instable ?

et déjà, ***quels bosons et fermions relier ?***

$$\left\{ \begin{array}{ccc} photon & \stackrel{?}{\longleftrightarrow} & neutrino \\ W^{\pm} & \stackrel{?}{\longleftrightarrow} & e^{\pm} \\ gluons & \stackrel{?}{\longleftrightarrow} & quarks \\ & \dots & \end{array} \right.$$

ne convient pas ...

LES SUPERPARTENAIRES

Mais à chaque particule connue pourrait être associée

une particule image, son **reflet par supersymétrie** :

$$\left\{\begin{array}{lcl} \textbf{\textit{photon}} & \leftrightarrow & \text{spin-}\tfrac{1}{2}\ \textbf{\textit{photino}} \\ \textbf{\textit{gluons}} & \leftrightarrow & \text{spin-}\tfrac{1}{2}\ \textbf{\textit{gluinos}} \end{array}\right. \qquad \left\{\begin{array}{lcl} \textbf{\textit{leptons}} & \leftrightarrow & \text{spin-0}\ \textbf{\textit{sleptons}} \\ \textbf{\textit{quarks}} & \leftrightarrow & \text{spin-0}\ \textbf{\textit{squarks}} \end{array}\right. \quad \ldots$$

alors:

bosons connus	$\longleftrightarrow$	**nouveaux fermions**
fermions connus	$\longleftrightarrow$	**nouveaux bosons**

($\rightarrow$ pas de relation directe entre les forces et particules connues ...)

longtemps moqué comme un signe de l'inutilité de la supersymétrie

mais maintenant considéré comme "évident" !

Les BOSONS de SPIN 0

squarks et sleptons: nouveaux bosons de spin-0

pas de la même nature que les bosons BEH

associés à la brisure électrofaible et à l'origine des masses

Distingués par le nouveau nombre quantique de R parité

(aussi à l'origine de la stabilité de la matière sombre)

$$\begin{cases} \text{squarks et sleptons:} & R\text{-parité } -1 \\ \text{bosons BEH (et autres):} & R\text{-parité } +1 \end{cases}$$

interactions quartiques des champs BEH = interactions de jauge électrofaibles

$$V_{\text{quartic}} = \frac{g^2 + g'^2}{8} \, (h_1^\dagger h_1 - h_2^\dagger h_2)^2 + \frac{g^2}{2} \, |h_1^\dagger h_2|^2$$

potential quartique du MSSM

Le MODÈLE STANDARD SUPERSYMÉTRIQUE

(1974-77)

(contenu minimal)

Spin 1	Spin 1/2	Spin 0
gluons g photon γ	gluinos $\tilde{g}$ photino $\tilde{\gamma}$	
$W^\pm$ Z	winos $\widetilde{W}^\pm_{1,2}$ zinos $\widetilde{Z}_{1,2}$ higgsino $\tilde{h}^0$	$\left.\begin{array}{c} H^\pm \\ h \\ H,\ A \end{array}\right\}$ bosons BEH
	leptons l quarks q	sleptons $\tilde{l}$ squarks $\tilde{q}$

4 neutralinos (au moins) qui se mélangent

le plus léger, stable $\rightarrow$ ***Matière Sombre (?)***

2 doublets $\Rightarrow$ ***5 bosons BEH****, au moins*

avec mélange H/h*, l'un d'eux à 125 GeV/*c^2

Le ***NEUTRALINO*** le plus léger

associé au photon, au Z, à un boson de Higgs (ou à tous à la fois)

doit être stable, par la symétrie de **R-parité**

$$\boldsymbol{R_p = (-1)^{2S}\,(-1)^{3B+L}}$$

associée à une réflexion de la coordonnée anticommutante $\boldsymbol{\theta}$

$$\boldsymbol{\theta \;\to\; -\,\theta}$$

candidat naturel pour la Matière Sombre de l'Univers

MATIÈRE SOMBRE reliée

aux médiateurs (γ et Z) des interactions

et/ou aux bosons BEH ?

$\Rightarrow$ recherche de **matière sombre** aux collisionneurs de particules ...

Mais où sont les PARTICULES SUPERSYMÉTRIQUES ... ?

toujours inobservées, même au LHC

expériences ATLAS, CMS $\Rightarrow$

Les gluinos et les squarks – s'ils existent – doivent être

plus lourds que $\approx$ $\mathbf{TeV}/c^2$ (dans la plupart des cas)

à suivre:

La montée en énergie du LHC, de 8 à 13 TeV

scénario optimiste: découverte ☺
(implications considérables ...)

scénario pessimiste:
- elles n'existent pas ☹
- ou sont encore trop lourdes ... 😐

La MUSIQUE des SPARTICULES ... ?

Les particules supersymétriques *pourraient* ***"vibrer"***

($\equiv$ sparticules, de R-parité -1)

le long de ***dimensions supplémentaires cachées ... ?***

extrêmement petites ($\lesssim\ \mathbf{10^{-17}}$ cm)

($\boldsymbol{R}$-parité alors associée au fait de parcourir un cycle le long d'une dimension supplémentaire)

Elles seraient alors très lourdes !!

($\boldsymbol{L} \lesssim \mathbf{10^{-17}}$ cm $\leftrightarrow$ $\boldsymbol{mc^2} \approx \frac{\boldsymbol{\pi\hbar c}}{\boldsymbol{L}} \gtrsim \mathbf{6}$ TeV)

Plus la dimension est petite, plus la vibration est aigüe, et plus la particule est lourde ...

$\Rightarrow$ très grandes masses $\approx \dfrac{\boldsymbol{\pi\hbar}}{\boldsymbol{Lc}} \gtrsim$ **quelques TeV$/c^2$** (ou même bien plus ... ??)

perdrait-on alors l'espoir de voir des signes de la supersymétrie ?

pas forcément ...

Des signes de SUPERSYMÉTRIE dans le secteur de B-E-H ... ?

Même si les particules supersymétriques demeurent invisibles:

deux doublets de spin 0, $\begin{pmatrix} h_1^0 \\ h_1^- \end{pmatrix}$, $\begin{pmatrix} h_2^+ \\ h_2^0 \end{pmatrix}$

$\Rightarrow$ $\boxed{\begin{array}{c} 2 \text{ bosons chargés } H^+,\ H^- \\ \text{et } 3 \text{ neutres } H,\ h,\ A \ \textit{(au moins)} \end{array}}$

auto-interactions fixées par les constantes de jauge électrofaibles $g,\ g'$

masses reliées à m_W *et* m_Z

Le boson BEH le plus léger ne doit pas être trop lourd ...

c'est une bonne chose pour la supersymétrie que :

1) *trouver un tel boson de spin 0*

2) *semble fondamental, plutôt que composé* $\Rightarrow$ ☺

3) *à 125 GeV, et pas plus ...*

Un BOSON h à 125 GeV/c^2 ?

MSSM usuel

avant brisure de SUSY: *pas de brisure électrofaible* ☹

après brisure de SUSY: $m_h \leq m_Z \cos\beta + \underbrace{\textit{grandes (?) corrections radiatives}}$

dépendant de $\boxed{m_{\tilde{t}}}$

(potentiellement problématique ...) 😐

N/nMSSM

singulet S avec interaction (superpotentiel) trilinéaire $\lambda\, H_1\, H_2\, S$

(introduit il y a 40 ans, et pas pour “coller” aux résultats expérimentaux ...)

avant brisure de SUSY: *brisure électrofaible* + $\boxed{m_h = m_Z}$ ☺ (nMSSM)

après brisure de SUSY:

beaucoup plus facile *d'avoir* ***le boson scalaire le plus léger à 125 GeV/c^2*** ☺

Mais on peut aller plus loin:

$$\boxed{\textit{“Gauge - BEH UNIFICATION”}}$$

associations entre bosons $W^{\pm}$, Z et bosons BEH de spin 0 :

$$\left\{ \begin{array}{c} \boxed{\textit{spin-1 } Z \quad \overset{SUSY}{\longleftrightarrow} \overset{SUSY}{\longleftrightarrow} \quad \textit{spin-0 BEH boson}} \\ \textit{et} \\ \textit{spin-1 } W^{\pm} \quad \overset{SUSY}{\longleftrightarrow} \overset{SUSY}{\longleftrightarrow} \quad \textit{spin-0 } H^{\pm} \end{array} \right.$$

avec aussi des inos de spin-$\frac{1}{2}$

*le **neutralino** le plus léger étant candidat pour la Matière Sombre de l'Univers*

*Les mêmes **superchamps** $W^{\pm}(x,\theta,\bar{\theta}),\ Z(x,\theta,\bar{\theta})$ peuvent décrire à la fois les bosons $W^{\pm}$et Z (de spin 1) et les bosons BEH (de spin 0) associés*

dans les notations usuelles le partenaire de spin-0 du Z est

$$\underline{z = \sqrt{2}\, Re\, (h_2^0 \sin\beta - \, h_1^0 \cos\beta)}\,, \qquad \underline{\textit{voisin de } h \textit{ à grand } \tan\beta}$$

comportement très voisin de celui d'un H du modèle Standard !!

"Gauge-BEH unification" *(1974)*

Relie des particules alors inconnues, par une symétrie hypothétique !!

(quand peu de physiciens prenaient vraiment au sérieux l'existence d'un boson BEH ...)

---

40 ans après, la supersymétrie reste hypothétique, mais les particules sont là ...

Z *(1983)* h *(2012)*

Possibilité d'interpréter (à un angle de mélange près, éventuellement petit)

le **boson BEH à 125 GeV** *comme* **un Z dépourvu de spin**
relié au Z par deux transformations de supersymétrie

(à discuter, selon les résultats des expériences, les propriétés de ce boson, l'angle de mélange et le mécanisme de brisure de supersymétrie)

Un premier signe de la supersymétrie ?

EPJC 74 (2014) 2837 (arXiv:1403.5951) *PRD 90 (2014) 015033 (arXiv:1406.0093)*

http://www.refletsdelaphysique.fr/articles/refdp/pdf/1993/04/refdp_bsfp-91.pdf

[Sources et évolution de la physique quantique. Textes fondateurs (Masson; EDP-Sciences)]

The Standard Model Higgs Boson

Part of the Lecture Particle Physics II, UvA Particle Physics Master 2013-2014

Date: October 2013
Lecturer: Ivo van Vulpen
Assistant: Ivan Angelozzi

Disclaimer

These are private notes to prepare for the lecture on the Higgs mechanism, part of the of lecture Particle Physics II. The first two sections almost entirely based on the book 'Quarks and Leptons' from the authors F. Halzen & A. Martin. The rest is a collection of material takes from publications and the documents listed below.

Material used to prepare lecture:

- o Quarks and Leptons, F. Halzen & A. Martin (main source)
- o Gauge Theories of the Strong, Weak and Electromagnetic Interactions, C. Quigg
- o Introduction to Elementary Particles, D. Griffiths
- o Lecture notes, Particle Physics 1, M. Merk

Introduction

On July 4^{th} 2012 the ATLAS and CMS experiments at CERN presented their results in the search for the Higgs boson. The data collected at the Large Hadron Collider (LHC) during the first run clearly indicated that a new particle had been observed: the illustrious and long sought after Higgs boson. The search for this particle was one of the main reasons the LHC was constructed as the Higgs boson. It is not 'just' a new particle in particle physics, but really forms one of the foundations of the (electroweak sector of the) Standard Model: it allows to give masses to both fermions and gauge bosons in a local gauge invariance theory, it is at the heart of electroweak unification, quark mixing etc.

The importance discovery was clear a bit more than a year later when, on October 8^{th} 2013, François Englert and Peter Higgs were awarded the Nobel prize in physics:
"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERNs Large Hadron Collider"

In these lectures we will discuss the basics of electroweak symmetry breaking and the role of the Higgs mechanism in the Standard Model in quite some detail. We build on the lectures Particle Physics I where the electroweak sector if the Standard Model was presented and will only briefly touch on Quantum Chromo Dynamics. The last lecture will be a presentation on the experimental search, all extracted properties, it's interpretation and an overview of the open questions and problems. Despite the remarkable experimental confirmation of the Standard Model, even with the Higgs boson present, it is not able to explain several observations like dark matter, the special role of gravity and the expansion of the universe. It is these 'irritating' open questions that make particle physicists believe that the Standard Model is only a simplification of a more complex underlying structure.

1 Symmetry breaking

After a review of the shortcomings of the model of electroweak interactions in the Standard Model, in this section we study the consequences of spontaneous symmetry breaking of (gauge) symmetries. We will do this in three steps of increasing complexity and focus on the principles of how symmetry breaking can be used to obtain massive gauge bosons by working out in full detail the breaking of a local U(1) gauge invariant model (QED) and give the photon a mass.

1.1 Problems in the Electroweak Model

The electroweak model, beautiful as it is, has some serious shortcomings.

1] Local $\mathbf{SU(2)_L \times U(1)_Y}$ gauge invariance forbids massive gauge bosons

In the theory of Quantum ElectroDynamics (QED) the requirement of local gauge invariance, i.e. the invariance of the Lagrangian under the transformation $\phi^{'} \to e^{i\alpha(x)}\phi$ plays a fundamental rôle. Invariance was achieved by replacing the partial derivative by a covariant derivative, $\partial_\mu \to \mathcal{D}_\mu = \partial_\mu - ieA_\mu$ and the introduction of a new vector field A with very specific transformation properties: $A^{'}_\mu \to A_\mu + \frac{1}{e}\partial_\mu\alpha$. This Lagrangian for a free particle then changed to:

$$\mathcal{L}_{\mathrm{QED}} \;=\; \mathcal{L}_{\mathrm{free}} + \mathcal{L}_{\mathrm{int}} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

which not only 'explained' the presence of a vector field in nature (the photon), but also automatically yields an interaction term $\mathcal{L}_{\mathrm{int}} = eJ^\mu A_\mu$ between the vector field and the particle as explained in detail in the lectures on the electroweak model. Under these symmetry requirements it is unfortunately not possible for a gauge boson to acquire a mass. In QED for example, a mass term for the photon, would not be allowed as such a term breaks gauge invariance:

$$\frac{1}{2}m_\gamma^2 A_\mu A^\mu = \frac{1}{2}m_\gamma^2(A_\mu + \frac{1}{e}\partial_\mu\alpha)(A^\mu + \frac{1}{e}\partial^\mu\alpha) \neq \frac{1}{2}m_\gamma^2 A_\mu A^\mu$$

The example using only U(1) and the mass of the photon might sounds strange as the photon is actually massless, but a similar argument holds in the electroweak model for the W and Z bosons, particles that we *know* are massive and make the weak force only present at very small distances.

2] Local $\mathbf{SU(2)_L \times U(1)_Y}$ gauge invariance forbids massive fermions

Just like in QED, invariance under local gauge transformations in the electroweak model requires introducing a covariant derivative of the form $D_\mu = \partial_\mu + ig\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu + ig'\frac{1}{2}YB_\mu$ introducing a weak current, J^{weak} and a different transformation for isospin singlets and doublets. A mass term for a fermion in the Lagrangian would be of the form $-m_f\bar{\psi}\psi$, but such terms in the Lagrangian are not allowed as they are *not* gauge invariant. This is clear

when we decompose the expression in helicity states:

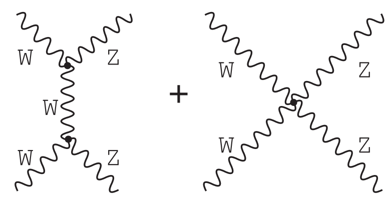
$$\begin{aligned} -m_f\bar{\psi}\psi &= -m_f\left(\bar{\psi_R}+\bar{\psi_L}\right)(\psi_L+\psi_R) \\ &= -m_f\left[\bar{\psi_R}\psi_L+\bar{\psi_L}\psi_R\right] \qquad \text{, since } \bar{\psi_R}\psi_R=\bar{\psi_L}\psi_L=0 \end{aligned}$$

Since ψ_L (left-handed, member of an isospin doublet, I $= \frac{1}{2}$) and ψ_R (right-handed, isospin singlet, I $= 0$) behave differently under rotations these terms are not gauge invariant:

$$\begin{aligned} {\psi_L}' \rightarrow \psi_L &= e^{i\alpha(x)T+i\beta(x)Y}\psi_L \\ {\psi_R}' \rightarrow \psi_R &= e^{i\beta(x)Y}\psi_R \end{aligned}$$

3] Violating unitarity

Several Standard Model scattering cross-sections, like WW-scattering (some Feynman graphs are shown in the picture on the right) violate unitarity at high energy as $\sigma(\mathrm{WW}\rightarrow\mathrm{ZZ}) \propto \mathrm{E}^2$. This energy dependency clearly makes the theory non-renormalizable.



How to solve the problems: a way out

To keep the theory renormalizable, we need a very high degree of symmetry (local gauge invariance) in the model. Dropping the requirement of the local $\mathrm{SU(2)_L \times U(1)_Y}$ gauge invariance is therefore not a wise decision. Fortunately there is a way out of this situation:

> Introduce a new field with a very specific potential that keeps the full Lagrangian invariant under $\mathrm{SU(2)_L \times U(1)_Y}$, but will make the vacuum *not* invariant under this symmetry. We will explore this idea, spontaneous symmetry breaking of a local gauge invariant theory (or Higgs mechanism), in detail in this section.

The Higgs mechanism: - Solves all the above problems
- Introduces a fundamental scalar $\rightarrow$ the Higgs boson !

1.2 A few basics on Lagrangians

A short recap of the basics on Lagrangians we'll be using later.

$$\mathcal{L} = \mathrm{T(kinetic)} - \mathrm{V(potential)}$$

The Euler-Lagrange equation then give you the equations of motion:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

For a real scalar field for example:

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2}\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right) - \frac{1}{2}m^2\phi^2 \rightarrow \text{Euler-Lagrange} \rightarrow \underbrace{(\partial_\mu\partial^\mu + m^2)\phi = 0}_{\text{Klein-Gordon equation}}$$

In electroweak theory, kinematics of fermions, i.e. spin-1/2 particles is described by:

$$\mathcal{L}_{\text{fermion}} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi \rightarrow \text{Euler-Lagrange} \rightarrow \underbrace{\left(i\gamma_\mu\partial^\mu - m\right)\psi = 0}_{\text{Dirac equation}}$$

In general, the Lagrangian for a real scalar particle (ϕ) is given by:

$$\mathcal{L} = \underbrace{(\partial_\mu\phi)^2}_{\text{kinetic term}} + \underbrace{C}_{\text{constant}} + \underbrace{\alpha\phi}_{?} + \underbrace{\beta\phi^2}_{\text{mass term}} + \underbrace{\gamma\phi^3}_{\text{3-point int.}} + \underbrace{\delta\phi^4}_{\text{4-point int.}} + \ ... \tag{1}$$

We can interpret the particle spectrum of the theory when studying the Lagrangian under small perturbations. In expression (1), the constant (potential) term is for most purposes of no importance as it does not appear in the equation of motion, the term linear in the field has no direct interpretation (and should not be present as we will explain later), the quadratic term in the fields represents the mass of the field/particle and higher order terms describe interaction terms.

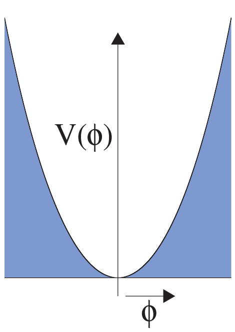
1.3 Simple example of symmetry breaking

To describe the main idea of symmetry breaking we start with a simple model for a real scalar field ϕ (or a theory to which we add a new field ϕ), with a specific potential term:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(\partial_\mu\phi)^2 - \mathrm{V}(\phi)\\ &= \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4\end{aligned} \tag{2}$$

Note that $\mathcal{L}$ is symmetric under $\phi \rightarrow -\phi$ and that λ is positive to ensure an absolute minimum in the Lagrangian. We can investigate in some detail the two possibilities for the sign of μ^2: positive or negative.

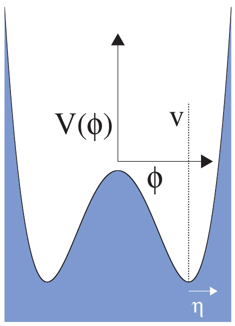
1.3.1 $\mu^2 > 0$: Free particle with additional interactions



To investigate the particle spectrum we look at the Lagrangian for small perturbations around the minimum (vacuum). The vacuum is at $\phi = 0$ and is symmetric in ϕ. Using expression (1) we see that the Lagrangian describes a free particle with mass μ that has an additional four-point self-interaction:

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}\mu^2\phi^2}_{\text{free particle, mass } \mu} \quad \underbrace{-\frac{1}{4}\lambda\phi^4}_{\text{interaction}}$$

1.3.2 $\mu^2 < 0$: Introducing a particle with imaginary mass ?



The situation with $\mu^2 < 0$ looks strange since at first glance it would appear to describe a particle ϕ with an imaginary mass. However, if we take a closer look at the potential, we see that it does not make sense to interpret the particle spectrum using the field ϕ since perturbation theory around $\phi = 0$ will not converge (not a stable minimum) as the vacuum is located at:

$$\phi_0 = \sqrt{-\frac{\mu^2}{\lambda}} = v \quad \text{or} \quad \mu^2 = -\lambda v^2 \tag{3}$$

As before, to investigate the particle spectrum in the theory, we have to look at small perturbations around this minimum. To do this it is more natural to introduce a field η (simply a shift of the ϕ field) that is centered at the vacuum: $\eta = \phi - v$.

Rewriting the Lagrangian in terms of η

Expressing the Lagrangian in terms of the shifted field η is done by replacing ϕ by $\eta + v$ in the original Lagrangian from equation (2):

$$\begin{aligned}\text{Kinetic term:} \quad \mathcal{L}_{\text{kin}}(\eta) &= \frac{1}{2}(\partial_\mu(\eta+v)\partial^\mu(\eta+v)) \\ &= \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) \qquad , \text{since } \partial_\mu v = 0.\end{aligned}$$

$$\begin{aligned}\text{Potential term:} \quad \mathrm{V}(\eta) &= +\frac{1}{2}\mu^2(\eta+v)^2 + \frac{1}{4}\lambda(\eta+v)^4 \\ &= \lambda v^2\eta^2 + \lambda v\eta^3 + \frac{1}{4}\lambda\eta^4 - \frac{1}{4}\lambda v^4,\end{aligned}$$

where we used $\mu^2 = -\lambda v^2$ from equation (3). Although the Lagrangian is still symmetric in ϕ, the perturbations around the minimum are *not* symmetric in η, i.e. $\mathrm{V}(-\eta) \neq \mathrm{V}(\eta)$. Neglecting the irrelevant $\frac{1}{4}\lambda v^4$ constant term and neglecting terms or order η^2 we have as Lagrangian:

$$\begin{aligned}\text{Full Lagrangian:} \quad \mathcal{L}(\eta) &= \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \lambda v^2\eta^2 - \lambda v\eta^3 - \frac{1}{4}\lambda\eta^4 - \frac{1}{4}\lambda v^4 \\ &= \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \lambda v^2\eta^2\end{aligned}$$

From section 1.2 we see that this describes the kinematics for a massive scalar particle:

$$\frac{1}{2}m_\eta^2 = \lambda v^2 \rightarrow m_\eta = \sqrt{2\lambda v^2} \quad \left(= \sqrt{-2\mu^2}\right) \qquad \text{Note: } m_\eta > 0.$$

Executive summary on $\mu^2 < 0$ scenario

At first glance, adding a $V(\phi)$ term as in equation (2) to the Lagrangian implies adding a particle with imaginary mass with a four-point self-interaction. However, when examining the particle spectrum using perturbations around the vacuum, we see that it actually describes a massive scalar particle (real, positive mass) with three- and four-point self-interactions. Although the Lagrangian retains its original symmetry (symmetric in ϕ), the vacuum is not symmetric in the field η: spontaneous symmetry breaking. Note that we have added a single degree of freedom to the theory: a scalar particle.

1.4 Breaking a global symmetry

In an existing theory we are free to introduce an additional complex scalar field: $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$ (two degrees of freedom):

$$\mathcal{L} = (\partial_\mu \phi)^*(\partial^\mu \phi) - \mathrm{V}(\phi) \qquad \text{, with } \mathrm{V}(\phi) = \mu^2(\phi^*\phi) + \lambda(\phi^*\phi)^2$$

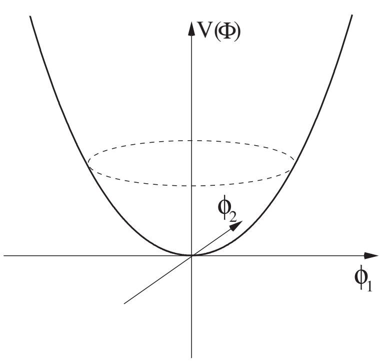
Note that the Lagrangian is invariant under a U(1) global symmetry, i.e. under $\phi' \to e^{i\alpha}\phi$ since $\phi'^*\phi' \to \phi^*\phi e^{-i\alpha}e^{+i\alpha} = \phi^*\phi$.

The Lagrangian in terms of ϕ_1 and ϕ_2 is given by:

$$\mathcal{L}(\phi_1, \phi_2) = \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 - \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2$$

There are again two distinct cases: $\mu^2 > 0$ and $\mu^2 < 0$. As in the previous section, we investigate the particle spectrum by studying the Lagrangian under small perturbations around the vacuum.

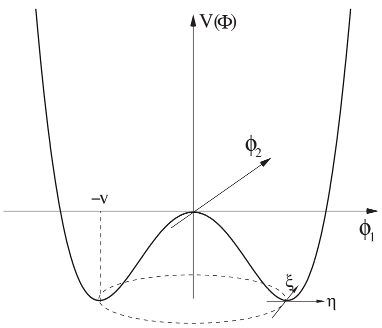
1.4.1 $\mu^2 > 0$



This situation simply describes two massive scalar particles, each with a mass μ with additional interactions:

$$\mathcal{L}(\phi_1, \phi_2) = \underbrace{\frac{1}{2}(\partial_\mu\phi_1)^2 - \frac{1}{2}\mu^2\phi_1^2}_{\text{particle } \phi_1\text{, mass } \mu} + \underbrace{\frac{1}{2}(\partial_\mu\phi_2)^2 - \frac{1}{2}\mu^2\phi_2^2}_{\text{particle } \phi_2\text{, mass } \mu} + \text{ interaction terms}$$

1.4.2 $\mu^2 < 0$

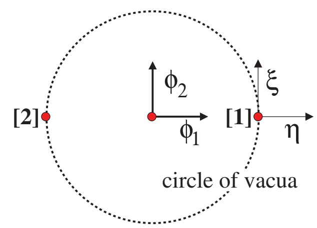


When $\mu^2 < 0$ there is not a single vacuum located at $\binom{0}{0}$, but an infinite number of vacua that satisfy:

$$\sqrt{\phi_1^2 + \phi_2^2} = \sqrt{\frac{-\mu^2}{\lambda}} = v$$

From the infinite number we choose ϕ_0 as $\phi_1 = v$ and $\phi_2 = 0$. To see what particles are present in this model, the behaviour of the Lagrangian is studied under small oscillations around the vacuum.

Looking at the symmetry we would use a $\alpha e^{i\beta}$. When looking at perturbations around this minimum it is natural to define the shifted fields η and ξ, with: $\eta = \phi_1 - v$ and $\xi = \phi_2$, which means that the (perturbations around the) vacuum are described by (see section 1.5.2):



$$\phi_0 = \frac{1}{\sqrt{2}}(\eta + v + i\xi)$$

Using $\phi^2 = \phi^*\phi = \frac{1}{2}[(v+\eta)^2 + \xi^2]$ and $\mu^2 = -\lambda v^2$ we can rewrite the Lagrangian in terms of the shifted fields.

$$\begin{aligned}
\text{Kinetic term:}\quad \mathcal{L}_{\text{kin}}(\eta,\xi) &= \frac{1}{2}\partial_\mu(\eta + v - i\xi)\partial^\mu(\eta + v + i\xi) \\
&= \frac{1}{2}(\partial_\mu\eta)^2 + \frac{1}{2}(\partial_\mu\xi)^2 \quad \text{, since } \partial_\mu v = 0.
\end{aligned}$$

$$\begin{aligned}
\text{Potential term:}\quad \mathrm{V}(\eta,\xi) &= \mu^2\phi^2 + \lambda\phi^4 \\
&= -\frac{1}{2}\lambda v^2[(v+\eta)^2 + \xi^2] + \frac{1}{4}\lambda[(v+\eta)^2 + \xi^2]^2 \\
&= -\frac{1}{4}\lambda v^4 + \lambda v^2\eta^2 + \lambda v\eta^3 + \frac{1}{4}\lambda\eta^4 + \frac{1}{4}\lambda\xi^4 + \lambda v\eta\xi^2 + \frac{1}{2}\lambda\eta^2\xi^2
\end{aligned}$$

Neglecting the constant and higher order terms, the full Lagrangian can be written as:

$$\mathcal{L}(\eta,\xi) = \underbrace{\frac{1}{2}(\partial_\mu\eta)^2 - (\lambda v^2)\eta^2}_{\text{massive scalar particle } \eta} + \underbrace{\frac{1}{2}(\partial_\mu\xi)^2 + 0\cdot\xi^2}_{\text{massless scalar particle } \xi} + \text{higher order terms}$$

We can identify this as a massive η particle and a massless ξ particle:

$$m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} > 0 \quad \text{and} \quad m_\xi = 0$$

Unlike the η-field, describing radial excitations, there is no 'force' acting on oscillations along the ξ-field. This is a direct consequence of the U(1) symmetry of the Lagrangian and the massless particle ξ is the so-called Goldstone boson.

Goldstone theorem:
For each broken generator of the original symmetry group, i.e. for each generator that connects the vacuum states one massless spin-zero particle will appear.

Executive summary on breaking a global gauge invariant symmetry

Spontaneously breaking a continuous global symmetry gives rise to a massless (Goldstone) boson. When we break a *local* gauge invariance something special happens and the Goldstone boson will disappear.

1.5 Breaking a local gauge invariant symmetry: the Higgs mechanism

In this section we will take the final step and study what happens if we break a *local* gauge invariant theory. As promised in the introduction, we will explore its consequences using a local U(1) gauge invariant theory we know (QED). As we will see, this will allow to add a mass-term for the gauge boson (the photon).

Local U(1) gauge invariance is the requirement that the Lagrangian is invariant under $\phi^{'} \to e^{i\alpha(x)}\phi$. From the lectures on electroweak theory we know that this can be achieved by switching to a covariant derivative with a special transformation rule for the vector field. In QED:

$$\begin{aligned} \partial_\mu &\to D_\mu = \partial_\mu - ieA_\mu && \text{[covariant derivatives]} \\ A_\mu^{'} &= A_\mu + \frac{1}{e}\partial_\mu\alpha && [A_\mu \text{ transformation}] \end{aligned} \tag{4}$$

The local U(1) gauge invariant Lagrangian for a complex scalar field is then given by:

$$\mathcal{L} = (D^\mu\phi)^\dagger (D_\mu\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)$$

The term $\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ is the kinetic term for the gauge field (photon) and $V(\phi)$ is the extra term in the Lagrangian we have seen before: $V(\phi^*\phi) = \mu^2(\phi^*\phi) + \lambda(\phi^*\phi)^2$.

1.5.1 Lagrangian under small perturbations

The situation $\mu^2 > 0$: we have a vacuum at $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. The exact symmetry of the Lagrangian is preserved in the vacuum: we have QED with a massless photon and two massive scalar particles ϕ_1 and ϕ_2 each with a mass μ.

In the situation $\mu^2 < 0$ we have an infinite number of vacua, each satisfying $\phi_1^2 + \phi_2^2 = -\mu^2/\lambda = v^2$. The particle spectrum is obtained by studying the Lagrangian under small oscillations using the same procedure as for the continuous global symmetry from section (1.4.2). Because of <u>local</u> gauge invariance some important differences appear. Extra terms will appear in the kinetic part of the Lagrangian through the covariant derivatives. Using again the shifted fields η and ξ we define the vacuum as $\phi_0 = \frac{1}{\sqrt{2}}[(v + \eta) + i\xi]$.

$$
\begin{aligned}
\text{Kinetic term:} \quad \mathcal{L}_{\text{kin}}(\eta,\xi) &= (D^\mu\phi)^\dagger\,(D_\mu\phi) \\
&= (\partial^\mu + ieA^\mu)\phi^*(\partial_\mu - ieA_\mu)\phi \\
&= \ \text{... see Exercise 1}
\end{aligned}
$$

$$
\text{Potential term:} \quad V(\eta,\xi) = \lambda v^2\eta^2 \text{ , up to second order in the fields. See section 1.4.2.}
$$

The full Lagrangian can be written as:

$$
\mathcal{L}(\eta,\xi) = \underbrace{\frac{1}{2}(\partial_\mu\eta)^2 - \lambda v^2\eta^2}_{\eta\text{-particle}} + \underbrace{\frac{1}{2}(\partial_\mu\xi)^2}_{\xi\text{-particle}} \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}e^2v^2A_\mu^2}_{\text{photon field}} - \underbrace{evA_\mu(\partial^\mu\xi)}_{?} + \text{int.-terms} \tag{5}
$$

At first glance: massive η, massless ξ (as before) and also a mass term for the photon. However, the Lagrangian also contains strange terms that we cannot easily interpret: $-evA_\mu(\partial^\mu\xi)$. This prevents making an easy interpretation.

1.5.2 Rewriting the Lagrangian in the unitary gauge

In a local gauge invariance theory we see that A_μ is fixed up to a term $\partial_\mu\alpha$ as can be seen from equation (4). In general, A_μ and ϕ change simultaneously. We can exploit this freedom, to redefine A_μ and remove all terms involving the ξ field.

Looking at the terms involving the ξ-field, we see that we can rewrite them as:

$$
\frac{1}{2}(\partial_\mu\xi)^2 - evA^\mu(\partial_\mu\xi) + \frac{1}{2}e^2v^2A_\mu^2 \ = \ \frac{1}{2}e^2v^2\left[A_\mu - \frac{1}{ev}(\partial_\mu\xi)\right]^2 \ = \ \frac{1}{2}e^2v^2(A_\mu^{'})^2
$$

This specific choice, i.e. taking $\alpha = -\xi/v$, is called the *unitary gauge*. Of course, when choosing this gauge (phase of rotation α) the field ϕ changes accordingly (see first part of section 1.1 and dropping terms of $\mathcal{O}(\xi^2,\eta^2,\xi\eta)$):

$$
\phi^{'} \to e^{-i\ \xi/v}\phi \ = \ e^{-i\ \xi/v}\frac{1}{\sqrt{2}}(v+\eta+i\xi) \ = \ e^{-i\ \xi/v}\frac{1}{\sqrt{2}}(v+\eta)e^{+i\ \xi/v} \ = \ \frac{1}{\sqrt{2}}(v+h)
$$

Here we have introduced the real h-field. When writing down the full Lagrangian in this specific gauge, we will see that all terms involving the ξ-field will disappear and that the additional degree of freedom will appear as the mass term for the gauge boson associated to the broken symmetry.

1.5.3 Lagrangian in the unitary gauge: particle spectrum

$$\begin{aligned}
\mathcal{L}_{\text{scalar}} &= (D^{\mu}\phi)^{\dagger}(D_{\mu}\phi) - V(\phi^{\dagger}\phi) \\
&= (\partial^{\mu} + ieA^{\mu})\frac{1}{\sqrt{2}}(v+h)\ \ (\partial_{\mu} - ieA_{\mu})\frac{1}{\sqrt{2}}(v+h) - V(\phi^{\dagger}\phi) \\
&= \frac{1}{2}(\partial_{\mu}h)^2 + \frac{1}{2}e^2A_{\mu}^2(v+h)^2 - \lambda v^2h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4 + \frac{1}{4}\lambda v^4
\end{aligned}$$

Expanding $(v+h)^2$ into 3 terms (and ignoring $\frac{1}{4}\lambda v^4$) we end up with:

$$= \underbrace{\frac{1}{2}(\partial_{\mu}h)^2 - \lambda v^2h^2}_{\text{massive scalar particle h}} + \underbrace{\frac{1}{2}e^2v^2A_{\mu}^2}_{\text{gauge field }(\gamma)\text{ with mass}} \underbrace{+\, e^2vA_{\mu}^2h + \frac{1}{2}e^2A_{\mu}^2h^2}_{\text{interaction Higgs and gauge fields}} \underbrace{-\, \lambda v h^3 - \frac{1}{4}\lambda h^4}_{\text{Higgs self-interactions}}$$

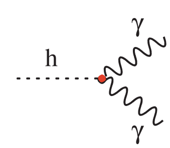
1.5.4 A few words on expanding the terms with $(\mathbf{v} + \mathbf{h})^2$

Expanding the terms in the Lagrangian associated to the vector field we see that we do not only get terms proportional to A_{μ}^2, i.e. a mass term for the gauge field (photon), but also automatically terms that describe the interaction of the Higgs field with the gauge field. These interactions, related to the mass of the gauge boson, are a consequence of the Higgs mechanism.

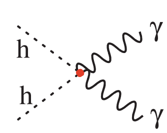
In our model, QED with a massive photon, when expanding $\frac{1}{2}e^2A_{\mu}^2(v+h)^2$ we get:

1] $\frac{1}{2}e^2v^2A_{\mu}^2$: the mass term for the gauge field (photon)
Given equation (1) we see that $m_{\gamma} = ev$.

2] $e^2vA_{\mu}^2h$: photon-Higgs three-point interaction



3] $\frac{1}{2}e^2A_{\mu}^2h^2$: photon-Higgs four-point interaction



Executive summary on breaking a local gauge invariant symmetry

We added a complex scalar field (2 degrees of freedom) to our existing theory and broke the original symmetry by using a 'strange' potential that yielded a large number of vacua. The additional degrees of freedom appear in the theory as a mass term for the gauge boson connected to the broken symmetry (m_{γ}) and a massive scalar particle (m_h).

$\boxed{\text{Exercises lecture 1}}$

Exercise 1: interaction terms

a) Compute the 'interaction terms' as given in equation (5).

b) Are the interaction terms symmetric in η and ξ ?

Exercise 2: Toy-model with a massive photon

a) Derive expression (14.58) in Halzen & Martin.
Hint: you can either do the full computation or, much less work, just insert $\phi = \frac{1}{\sqrt{2}}(v+h)$ in the Lagrangian and keep A_μ unchanged.

b) Show that in this model the Higgs boson can decay into two photons and that the coupling $h \to \gamma\gamma$ is proportional to m_γ.

c) Draw all Feynman vertices that are present in this model and show that Higgs three-point (self-)coupling, or $h \to hh$, is proportional to $\mathrm{m_h}$.

d) Higgs boson properties: how can you see from the Lagrangian that the Higgs boson is a scalar (spin 0) particle ? What defines the 'charge' of the Higgs boson ?

Exercise 3: the potential part: $\mathbf{V}(\phi^\dagger\phi)$

Use in this exercise $\phi = \frac{1}{\sqrt{2}}(v+h)$ and that ϕ is real (1 dimension).

a) The normal Higgs potential: $V(\phi^\dagger\phi) = \mu^2\phi^2 + \lambda\phi^4$
Show that $\frac{1}{2}m_h^2 = \lambda v^2$, where $(\phi_0 = v)$. How many vacua are there?

b) Why is $V(\phi^\dagger\phi) = \mu^2\phi^2 + \beta\phi^3$ not possible ?
How many vacua are there?

Terms $\propto \phi^6$ are allowed since they introduce additional interactions that are not cancelled by gauge boson interactions, making the model non-renormalizable. Just ignore this little detail for the moment and compute the 'prediction' for the Higgs boson mass.

c) Use $V(\phi^\dagger\phi) = \mu^2\phi^2 - \lambda\phi^4 + \frac{4}{3}\delta\phi^6$, with $\mu^2 < 0$, $\lambda > 0$ and $\delta = -\frac{2\lambda^2}{\mu^2}$.
Show that $\mathrm{m}_h(\text{new}) = \sqrt{\frac{3}{2}}\mathrm{m}_h(\text{old})$, with 'old': m_h for the normal Higgs potential.

2 The Higgs mechanism in the Standard Model

In this section we will apply the idea of spontaneous symmetry breaking from section 1 to the model of electroweak interactions. With a specific choice of parameters we can obtain massive Z and W bosons while keeping the photon massless.

2.1 Breaking the local gauge invariant $\mathbf{SU(2)_L \times U(1)_Y}$ symmetry

To break the $\mathrm{SU(2)_L \times U(1)_Y}$ symmetry we follow the ingredients of the Higgs mechanism:

1) Add an isospin doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

Since we would like the Lagrangian to retain all its symmetries, we can only add $\mathrm{SU(2)_L \times U(1)_Y}$ multiplets. Here we add a left-handed doublet (like the electron neutrino doublet) with weak Isospin $\frac{1}{2}$. The electric charges of the upper and lower component of the doublet are chosen to ensure that the hypercharge Y=+1. This requirement is vital for reasons that will become more evident later.

2) Add a potential V(ϕ) for the field that will break (spontaneously) the symmetry:

$$V(\phi) = \mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2 \quad \text{, with } \mu^2 < 0$$

The part added to the Lagrangian for the scalar field

$$\mathcal{L}_{\text{scalar}} = (D^\mu\phi)^\dagger(D_\mu\phi) - V(\phi),$$

where D_μ is the covariant derivative associated to $\mathrm{SU(2)_L \times U(1)_Y}$:

$$D_\mu = \partial_\mu + ig\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu + ig'\frac{1}{2}YB_\mu$$

3) Choose a vacuum:
We have seen that any choice of the vacuum that breaks a symmetry will generate a mass for the corresponding gauge boson. The vacuum we choose has ϕ_1=ϕ_2=ϕ_4=0 and $\phi_3 = v$:

$$\text{Vacuum} = \phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

This vacuum as defined above is neutral since $I = \frac{1}{2}$, $I_3 = -\frac{1}{2}$ and with our choice of $Y = +1$ we have $Q = I_3 + \frac{1}{2}Y$=0. We will see that this choice of the vacuum breaks $\mathrm{SU(2)_L \times U(1)_Y}$,but leaves $\mathrm{U(1)_{EM}}$ invariant, leaving only the photon massless. In writing down this vacuum we immediately went to the unitary gauge (see section 1.5).

2.2 Checking which symmetries are broken in a given vacuum

How do we check if the symmetries associated to the gauge bosons are broken ? Invariance implies that $e^{i\alpha Z}\phi_0 = \phi_0$, with Z the associated 'rotation'. Under infinitesimal rotations this means $(1 + i\alpha Z)\phi_0 = \phi_0 \rightarrow Z\phi_0 = 0$.

What about the $\mathrm{SU(2)_L}$, $\mathrm{U(1)_Y}$ and $\mathrm{U(1)_{EM}}$ generators:

$$\begin{aligned}
\mathrm{SU(2)_L}: \quad \tau_1\phi_0 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} = +\frac{1}{\sqrt{2}} \begin{pmatrix} v+h \\ 0 \end{pmatrix} \neq 0 \rightarrow \text{broken} \\
\tau_2\phi_0 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} = -\frac{i}{\sqrt{2}} \begin{pmatrix} v+h \\ 0 \end{pmatrix} \neq 0 \rightarrow \text{broken} \\
\tau_3\phi_0 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} = -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} \neq 0 \rightarrow \text{broken} \\
\mathrm{U(1)_Y}: \quad Y\phi_0 &= Y_{\phi_0} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} = +\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} \neq 0 \rightarrow \text{broken}
\end{aligned}$$

This means that all 4 gauge bosons (W_1, W_2, W_3 and B) acquire a mass through the Higgs mechanism. In the lecture on electroweak theory we have seen that the W_1 and W_2 fields mix to form the charged W^+ and W^- bosons and that the W_3 and B field will mix to form the neutral Z-boson and photon.

$$\underbrace{W_1 \quad W_2}_{\text{W}^+ \text{ and W}^- \text{ bosons}} \qquad \underbrace{W_3 \quad B}_{\text{Z-boson and } \gamma}$$

When computing the masses of these mixed physical states in the next sections, we will see that one of these combinations (the photon) remains massless. Looking at the symmetries we can already predict this is the case. For the photon to remain massless the $\mathrm{U(1)_{EM}}$ symmetry should leave the vacuum invariant. And indeed:

$$\mathrm{U(1)_{EM}}: \quad Q\phi_0 = \frac{1}{2}(\tau_3 + Y)\phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} = 0 \rightarrow \text{unbroken}$$

It is not so strange that $\mathrm{U(1)_{EM}}$ is conserved as the vacuum is neutral and we have:

$$\phi_0' \rightarrow e^{i\alpha Q_{\phi_0}}\phi_0 = \phi_0$$

Breaking of $\mathbf{SU(2)_L \times U(1)_Y}$: looking a bit ahead

1) $\mathrm{W_1}$ and $\mathrm{W_2}$ mix and will form the massive a $\mathrm{W^+}$ and $\mathrm{W^-}$ bosons.
2) $\mathrm{W_3}$ and B mix to form massive Z and massless γ.
3) Remaining degree of freedom will form the mass of the scalar particle (Higgs boson).

2.3 Scalar part of the Lagrangian: gauge boson mass terms

Studying the scalar part of the Lagrangian

To obtain the masses for the gauge bosons we will only need to study the scalar part of the Lagrangian:

$$\mathcal{L}_{\text{scalar}} = (D^\mu\phi)^\dagger(D_\mu\phi) - V(\phi) \tag{6}$$

The $V(\phi)$ term will again give the mass term for the Higgs boson and the Higgs self-interactions. The $(D^\mu\phi)^\dagger(D_\mu\phi)$ terms:

$$D_\mu\phi = \left[\partial_\mu + ig\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu + ig'\frac{1}{2}YB_\mu\right]\ \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+h\end{pmatrix}$$

will give rise to the masses of the gauge bosons (and the interaction of the gauge bosons with the Higgs boson) since, as we discussed in section 1.5.4, working out the $(v+h)^2$-terms from equation (6) will give us three terms:

1) Masses for the gauge bosons ($\propto v^2$)
2) Interactions gauge bosons and the Higgs ($\propto vh$) and ($\propto h^2$)

In the exercises we will study the interactions of the Higgs boson and the gauge boson (the terms in 2)) in detail, but since we are here primarily interested in the masses of the vector bosons we will only focus on 1):

$$\begin{aligned}
(D_\mu\phi) &= \frac{1}{\sqrt{2}}\left[ig\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu + ig'\frac{1}{2}YB_\mu\right]\begin{pmatrix}0\\ v\end{pmatrix}\\
&= \frac{i}{\sqrt{8}}\left[g(\tau_1W_1+\tau_2W_2+\tau_3W_3) + g^{'}YB_\mu\right]\begin{pmatrix}0\\ v\end{pmatrix}\\
&= \frac{i}{\sqrt{8}}\left[g\left(\begin{pmatrix}0 & W_1\\ W_1 & 0\end{pmatrix} + \begin{pmatrix}0 & -iW_2\\ iW_2 & 0\end{pmatrix} + \begin{pmatrix}W_3 & 0\\ 0 & -W_3\end{pmatrix}\right) + g^{'}\begin{pmatrix}Y_{\phi_0}B_\mu & 0\\ 0 & Y_{\phi_0}B_\mu\end{pmatrix}\right]\begin{pmatrix}0\\ v\end{pmatrix}\\
&= \frac{i}{\sqrt{8}}\begin{pmatrix}gW_3+g^{'}Y_{\phi_0}B_\mu & g(W_1-iW_2)\\ g(W_1+iW_2) & -gW_3+g^{'}Y_{\phi_0}B_\mu\end{pmatrix}\begin{pmatrix}0\\ v\end{pmatrix}\\
&= \frac{iv}{\sqrt{8}}\begin{pmatrix}g(W_1-iW_2)\\ -gW_3+g^{'}Y_{\phi_0}B_\mu\end{pmatrix}
\end{aligned}$$

We can then also easily compute $(D^\mu\phi)^\dagger$: $(D^\mu\phi)^\dagger = -\frac{iv}{\sqrt{8}}\left(g(W_1+iW_2)\ ,\ (-gW_3+g^{'}Y_{\phi_0}B_\mu)\right)$ and we get the following expression for the kinetic part of the Lagrangian:

$$(D^\mu\phi)^\dagger(D_\mu\phi) = \frac{1}{8}v^2\left[g^2(W_1^2+W_2^2) + (-gW_3+g^{'}Y_{\phi_0}B_\mu)^2\right] \tag{7}$$

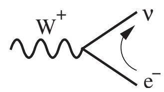
2.3.1 Rewriting $(\mathbf{D}^\mu\phi)^\dagger(\mathbf{D}_\mu\phi)$ in terms of physical gauge bosons

Before we can interpret this we need to rewrite this in terms of W$^+$, W$^-$, Z and γ since that are the gauge bosons that are observed in nature.

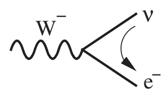
1] Rewriting terms with $\mathbf{W_1}$ and $\mathbf{W_2}$ terms: charged gauge bosons $\mathbf{W^+}$ and $\mathbf{W^-}$

When discussing the charged current interaction on $\mathrm{SU(2)_L}$ doublets we saw that the charge raising and lowering operators connecting the members of isospin doublets were τ_+ and τ_-, linear combinations of τ_1 and τ_2 and that each had an associated gauge boson: the W^+ and W^-.

$$\tau_+ = \frac{1}{2}(\tau_1 + i\tau_2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$



$$\tau_- = \frac{1}{2}(\tau_1 - i\tau_2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$



We can rewrite W_1, W_2 terms as W^+, W^- using $W^\pm = \frac{1}{\sqrt{2}}(W_1 \mp iW_2)$. In particular, $\frac{1}{2}(\tau_1 W_1 + \tau_2 W_2) = \frac{1}{\sqrt{2}}(\tau_+ W^+ + \tau_- W^-)$.

Looking at the terms involving W_1 and W_2 in the Lagrangian in equation (7), we see that:

$$g^2(W_1^2 + W_2^2) = g^2({W^+}^2 + {W^-}^2) \text{ or, alternatively, } 2g^2 W^+ W^- \tag{8}$$

2] Rewriting terms with $\mathbf{W_3}$ and $\mathbf{B_\mu}$ terms: neutral gauge bosons Z and γ

$$(-gW_3 + g'Y_{\phi_0}B_\mu)^2 = (W_3, B_\mu)\begin{pmatrix} g^2 & -gg'Y_{\phi_0} \\ -gg'Y_{\phi_0} & g'^2 \end{pmatrix}\begin{pmatrix} W_3 \\ B_\mu \end{pmatrix}$$

When looking at this expression there are some important things to note, especially related to the role of the hypercharge of the vacuum, Y_{ϕ_0}:

1 Only if $Y_{\phi_0} \neq 0$, the W_3 and B_μ fields mix.

2 If $Y_{\phi_0} = \pm\, 1$, the determinant of the mixing matrix vanishes and one of the combinations will be massless (the coefficient for that gauge field squared is 0). In our choice of vacuum we have $Y_{\phi_0} = +1$ (see Exercise 4 why that is a good idea). In the rest of our discussion we will drop the term Y_{ϕ_0} and simply use its value of 1.

The two eigenvalues and eigenvectors are given by [see Exercise 3]:

eigenvalue		*eigenvector*		
$\lambda = 0$	$\rightarrow$	$\frac{1}{\sqrt{g^2+g'^2}}\begin{pmatrix} g' \\ g \end{pmatrix} = \frac{1}{\sqrt{g^2+g'^2}}(g'W_3 + gB_\mu) = A_\mu$		photon(γ)
$\lambda = (g^2 + g'^2)$	$\rightarrow$	$\frac{1}{\sqrt{g^2+g'^2}}\begin{pmatrix} g \\ -g' \end{pmatrix} = \frac{1}{\sqrt{g^2+g'^2}}(gW_3 - g'B_\mu) = Z_\mu$		Z-boson (Z)

Looking at the terms involving W_3 and B in the Lagrangian we see that:

$$(-gW_3 + g'Y_{\phi_0}B_\mu)^2 = (g^2 + g'^2)Z_\mu^2 + 0 \cdot A_\mu^2 \tag{9}$$

3] Rewriting Lagrangian in terms of physical fields: masses of the gauge bosons

Finally, by combining equation (8) and (9) we can rewrite the Lagrangian from equation (7) in terms of the physical gauge bosons:

$$(D^{\mu}\phi)^{\dagger}(D_{\mu}\phi) = \frac{1}{8}v^2[g^2(W^+)^2 + g^2(W^-)^2 + (g^2 + g'^2)Z_{\mu}^2 + 0 \cdot A_{\mu}^2] \tag{10}$$

2.4 Masses of the gauge bosons

2.4.1 Massive charged and neutral gauge bosons

As a general mass term for a massive gauge boson V has the form $\frac{1}{2}M_V^2V_{\mu}^2$, from equation (10) we see that:

$$\begin{aligned} M_{W^+} = M_{W^-} &= \frac{1}{2}vg \\ M_Z &= \frac{1}{2}v\sqrt{(g^2 + g'^2)} \end{aligned}$$

Although since g and g' are free parameters, the SM makes no absolute predictions for $\mathrm{M_W}$ and $\mathrm{M_Z}$, it has been possible to set a lower limit before the W- and Z-boson were discovered (see Exercise 2). The measured values are $M_W = 80.4$ GeV and $M_Z = 91.2$ GeV.

Mass relation W and Z boson:

Although there is no absolute prediction for the mass of the W- and Z-boson, there is a clear prediction on the ratio between the two masses. From discussions in QED we know the photon couples to charge, which allowed us to relate e, g and g' (see Exercise 3):

$$e = g\sin(\theta_{\mathrm{W}}) = g'\cos(\theta_{\mathrm{W}}) \tag{11}$$

In this expression θ_{W} is the Weinberg angle, often used to describe the mixing of the W_3 and B_{μ} -fields to form the physical Z boson and photon. From equation (11) we see that $g'/g = \tan(\theta_{\mathrm{W}})$ and therefore:

$$\frac{M_W}{M_Z} = \frac{\frac{1}{2}vg}{\frac{1}{2}v\sqrt{g^2 + g'^2}} = \cos(\theta_{\mathrm{W}})$$

This predicted ratio is often expressed as the so-called ρ-(Veltman) parameter:

$$\rho = \frac{M_W^2}{M_Z^2\cos^2(\theta_W)} = 1$$

The current measurements of the M_W, M_Z and θ_{W} confirm this relation.

2.4.2 Massless neutral gauge boson (γ):

Similar to the Z boson we have now a mass for the photon: $\frac{1}{2}M_{\gamma}^2 = 0$, so:

$$M_{\gamma} = 0. \tag{12}$$

2.5 Mass of the Higgs boson

Looking at the mass term for the scalar particle, the mass of the Higgs boson is given by:

$$m_h = \sqrt{2\lambda v^2}$$

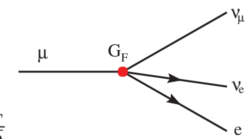
Although v is known ($v \approx 246$ GeV, see below), since λ is a free parameter, the mass of the Higgs boson is **not** predicted in the Standard Model.

Extra: how do we know v ?:

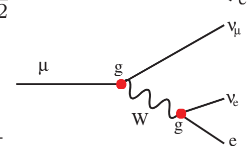
$$\text{Muon decay: } \frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}} \rightarrow v = \sqrt{\frac{1}{\sqrt{2}G_F}}$$

We used $M_W = \frac{1}{2}vg$. Given $G_F = 1.166 \cdot 10^{-5}$, we see that $v = 246$ GeV. This energy scale is known as the electroweak scale.

Fermi: $\propto \frac{G_F}{\sqrt{2}}$



EW: $\propto \frac{g^2}{8M_W^2}$



Exercises lecture 2

Exercise [1]: Higgs - Vector boson couplings

In the lecture notes we focussed on the masses of the gauge bosons, i.e. part 1) when expanding the $((v+h)^2)$-terms as discussed in Section 1.5.4 and 2.3. Looking now at the terms in the Lagrangian that describe the interaction between the gauge fields and the Higgs field, show that the four vertex factors describing the interaction between the Higgs boson and gauge bosons: hWW, hhWW, hZZ, hhZZ are given by:

3-point: $2i\frac{M_V^2}{v}g^{\mu\nu}$ and 4-point: $2i\frac{M_V^2}{v^2}g^{\mu\nu}$, with (V= W,Z).

Note: A vertex factor is obtained by multiplying the term involving the interacting fields in the Lagrangian by a factor i and a factor $n!$ for n identical particles in the vertex.

Exercise [2]: History: lower limits on $\mathrm{M_W}$ and $\mathrm{M_Z}$

Use the relations $e = g\sin\theta_W$ and $G_F = (v^2\sqrt{2})^{-1}$ to obtain lower limits for the masses of the W and Z boson assuming that you do not know the value of the weak mixing angle.

Exercise [3]: Electroweak mixing: $(\mathbf{W^3_\mu}, \mathbf{B_\mu}) \to (\mathbf{A_\mu}, \mathbf{Z_\mu})$.

The mix between the W^3_μ and B_μ fields in the lagrangian can be written in a matrix notation:

$$(W^3_\mu, B_\mu) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}$$

a) Show that the eigenvalues of the matrix are $\lambda_1 = 0$ and $\lambda_2 = (g^2 + g'^2)$.

b) Show that these eigenvalues correspond to the two eigenvectors:

$$V_1 = \frac{1}{\sqrt{g^2+g'^2}}(g'W^3_\mu + gB_\mu) \equiv \mathrm{A}_\mu \quad \text{and} \quad V_2 = \frac{1}{\sqrt{g^2+g'^2}}(gW^3_\mu - g'B_\mu) \equiv \mathrm{Z}_\mu$$

c) **bonus:** Imagine that we would have chosen $Y_{\phi_0'} = -1$. What, in that scenario, would be the (mass-)eigenvectors A'_μ and Z'_μ, the 'photon' and 'Z-boson' ? In such a model, what would be their masses ? Compare them to those in the Standard Model.

Exercise [4]: A closer look at the covariant derivative

The covariant derivative in the electroweak theory is given by:

$$D_\mu = \partial_\mu + ig'\frac{Y}{2}B_\mu + ig\vec{T}\cdot\vec{W}_\mu$$

Looking only at the part involving W^3_μ and B_μ show that:

$$D_\mu = \partial_\mu + i\mathrm{A}_\mu \frac{gg'}{\sqrt{g'^2+g^2}}\left(T_3 + \frac{Y}{2}\right) + i\mathrm{Z}_\mu \frac{1}{\sqrt{g'^2+g^2}}\left(g^2T_3 - g'^2\frac{Y}{2}\right)$$

Make also a final interpretation step for the A_μ part and show that:

$$\frac{gg'}{\sqrt{g'^2+g^2}} = e \quad \text{and} \quad T_3 + \frac{Y}{2} = Q, \text{ the electric charge.}$$

c) **bonus:** Imagine that we would have chosen $Y_{\phi_0'} = -1$. Show explicitly that in that case the photon does not couple to the electric charge.

Exercise [5] Gauge bosons in a model with an $\mathbf{SU(2)_L}$ symmetry

Imagine a system described by a local $\mathrm{SU(2)_L}$ gauge symmetry (iso-spin only) in which all gauge bosons are be massive. Note that this is different from the $\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$ symmetry of the SM involving also hypercharge. In this alternative model:

a) Explain why the Higgs field ϕ needs to be an $\mathrm{SU(2)_L}$ doublet.
b) How many gauge bosons are there and how many degrees of freedom does ϕ have ?
c) Determine the masses of the gauge bosons in this model.
d) What property of the particles do the gauge bosons couple to and what defines the 'charge' of the gauge bosons themselves ?

3 Fermion masses, Higgs decay and limits on $\mathrm{m_h}$

In this section we discuss how fermions acquire a mass and use our knowledge on the Higgs coupling to fermions and gauge bosons to predict how the Higgs boson decays as a function of its mass. Even though the Higgs boson has been discovered, we also discuss what theoretical information we have on the mass of the Higgs boson as it reveals the impact on the Higgs boson at higher energy scales (evolution of the universe).

3.1 Fermion masses

In section 1 we saw that terms like $\frac{1}{2}B_\mu B^\mu$ and $m\bar{\psi}\psi$ were not gauge invariant. Since these terms are not allowed in the Lagrangian, both gauge bosons and fermions are massless. In the previous section we have seen how the Higgs mechanism can be used to accommodate massive gauge bosons in our theory while keeping the local gauge invariance. As we will now see, the Higgs mechanism can also give fermions a mass: 'twee vliegen in een klap'.

Chirality and a closer look at terms like $-m\bar{\psi}\psi$

A term like $-m\bar{\psi}\psi = -m[\bar{\psi_L}\psi_R + \bar{\psi_R}\psi_L]$, i.e. a decomposition in chiral states (see exercise 1). Such a term in the Lagrangian is <u>not</u> gauge invariant since the left handed fermions form an isospin doublet (for example $\begin{pmatrix} \nu \\ e \end{pmatrix}_L$) and the right handed fermions form isospin singlets like e_R. They transform differently under $\mathrm{SU(2)_L \times U(1)_Y}$.

$$
\begin{aligned}
\text{left handed doublet} &= \chi_L \to \chi_L^{'} = \chi_L e^{i\vec{W}\cdot\vec{T}+i\alpha Y} \\
\text{right handed singlet} &= \psi_R \to \psi_R^{'} = \psi_R e^{i\alpha Y}
\end{aligned}
$$

This means that the term is not invariant under all $\mathrm{SU(2)_L \times U(1)_Y}$ 'rotations'.

Constructing an $\mathrm{SU(2)_L \times U(1)_Y}$ invariant term for fermions

If we could make a term in the Lagrangian that is a *singlet* under $\mathrm{SU(2)_L}$ and $\mathrm{U(1)_Y}$, it would remain invariant. This can be done using the complex (Higgs) doublet we introduced in the previous section. It can be shown that the Higgs has exactly the right quantum numbers to form an $SU(2)_L$ and $U(1)_Y$ singlet in the vertex: $-\lambda_f\bar{\psi}_L\phi\psi_R$, where λ_f is a so-called Yukawa coupling.

Executive summary: - a term: $\propto \bar{\psi}_L\psi_R$ is **not** invariant under $\mathrm{SU(2)_L \times U(1)_Y}$
- a term: $\propto \bar{\psi}_L\phi\psi_R$ is invariant under $\mathrm{SU(2)_L \times U(1)_Y}$

We have constructed a term in the Lagrangian that couples the Higgs doublet to the fermion fields:

$$
\mathcal{L}_{\text{fermion-mass}} = -\lambda_f[\bar{\psi}_L\phi\psi_R + \bar{\psi}_R\bar{\phi}\psi_L] \tag{13}
$$

When we write out this term we'll see that this does not only describe an interaction between the Higgs field and fermion, but that the fermions will acquire a finite mass if the ϕ-doublet has a non-zero expectation value. This is the case as $\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$ as before.

3.1.1 Lepton masses

$$
\begin{aligned}
\mathcal{L}_e &= -\lambda_e \frac{1}{\sqrt{2}} \left[(\bar{\nu}, \bar{e})_L \begin{pmatrix} 0 \\ v+h \end{pmatrix} e_R + \bar{e}_R (0, v+h) \begin{pmatrix} \nu \\ e \end{pmatrix}_L \right] \\
&= -\frac{\lambda_e (v+h)}{\sqrt{2}} [\bar{e}_L e_R + \bar{e}_R e_L] \\
&= -\frac{\lambda_e (v+h)}{\sqrt{2}} \bar{e} e \\
&= -\underbrace{\frac{\lambda_e v}{\sqrt{2}} \bar{e} e}_{\text{electron mass term}} - \underbrace{\frac{\lambda_e}{\sqrt{2}} h \bar{e} e}_{\text{electron-higgs interaction}} \\
& \qquad m_e = \frac{\lambda_e v}{\sqrt{2}} \qquad\qquad \frac{\lambda_e}{\sqrt{2}} \propto m_e
\end{aligned}
$$

A few side-remarks:

1) The Yukawa coupling is often expressed as $\lambda_f = \sqrt{2}\left(\frac{m_f}{v}\right)$ and the coupling of the fermion to the Higgs field is $\frac{\lambda_f}{\sqrt{2}} = \frac{m_f}{v}$, so proportional to the mass of the fermion.

2) The mass of the electron is **not** predicted since λ_e is a free parameter. In that sense the Higgs mechanism does not say anything about the electron mass itself.

3) The coupling of the Higgs boson to electrons is **very** small:
The coupling of the Higgs boson to an electron-pair ($\propto \frac{m_e}{v} = \frac{g m_e}{2M_W}$) is very small compared to the coupling of the Higgs boson to a pair of W-bosons ($\propto g M_W$).

$$
\frac{\Gamma(h \to ee)}{\Gamma(h \to WW)} \propto \frac{\lambda^2_{eeh}}{\lambda^2_{WWh}} = \left(\frac{g m_e / 2M_W}{g M_W} \right)^2 = \frac{m_e^2}{4 M_W^4} \approx 1.5 \cdot 10^{-21}
$$

3.1.2 Quark masses

The fermion mass term $\mathcal{L}_{\text{down}} = \lambda_f \bar{\psi}_L \phi \psi_R$ (leaving out the hermitian conjugate term $\bar{\psi}_R \bar{\phi} \psi_L$ for clarity) only gives mass to 'down' type fermions, i.e. only to one of the isospin doublet components. To give the neutrino a mass and give mass to the 'up' type quarks (u, c, t), we need another term in the Lagrangian. Luckily it is possible to compose a new term in the Lagrangian, using again the complex (Higgs) doublet in combination with the fermion fields, that is gauge invariant under $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ and gives a mass to the up-type quarks. The mass-term for the up-type fermions takes the form:

$$
\mathcal{L}_{\text{up}} = \bar{\chi}_L \tilde{\phi}^c \phi_R + \text{h.c., with}
$$

$$
\tilde{\phi}^c = -i \tau_2 \phi^* = -\frac{1}{\sqrt{2}} \begin{pmatrix} (v+h) \\ 0 \end{pmatrix} \tag{14}
$$

Mass terms for fermions (leaving out h.c. term):

$$\begin{aligned}
\text{down-type:}\quad \lambda_d(\bar{u}_L,\bar{d}_L)\phi d_R &= \lambda_d(\bar{u}_L,\bar{d}_L)\begin{pmatrix}0\\ v\end{pmatrix} d_R = \lambda_d v\ \bar{d}_L d_R \\
\text{up-type:}\quad \lambda_u(\bar{u}_L,\bar{d}_L)\tilde{\phi}^c d_R &= \lambda_u(\bar{u}_L,\bar{d}_L)\begin{pmatrix}v\\ 0\end{pmatrix} u_R = \lambda_u v\ \bar{u}_L u_R
\end{aligned}$$

As we will discuss now, this is not the whole story. If we look more closely we'll see that we can construct more fermion-mass-type terms in the Lagrangian that cannot easily be interpreted. Getting rid of these terms is at the origin of quark mixing.

3.2 Yukawa couplings and the origin of Quark Mixing

This section will discuss in full detail the consequences of all possible allowed quark 'mass-like' terms and study the link between the Yukawa couplings and quark mixing in the Standard Model: the difference between *mass eigenstates* and *flavour eigenstates*.
If we focus on the part of the SM Lagrangian that describes the dynamics of spinor (fermion) fields ψ, the kinetic terms, we see that:

$$\mathcal{L}_{\text{kinetic}} = i\bar{\psi}(\partial^\mu\gamma_\mu)\psi,$$

where $\bar{\psi} \equiv \psi^\dagger\gamma^0$ and the spinor fields ψ. It is instructive to realise that the spinor fields ψ are the three fermion generations can be written in the following five (interaction) representations:

general spinor field	Ψ^I(color, weak iso-spin, hypercharge)
1) left handed quarks	$Q^I_{Li}(3,2,+1/3)$
2) right handed up-type quarks	$u^I_{Ri}(3,1,+4/3)$
3) right handed down-type quarks	$d^I_{Ri}(3,1,+1/3)$
4) left handed fermions	$L^I_{Li}(1,2,-1)$
5) right handed fermions	$l^I_{Ri}(1,1,-2)$

In this notation, $Q^I_{Li}(3,2,+1/3)$ describes an $\mathrm{SU(3)_C}$ triplet, $\mathrm{SU(2)_L}$ doublet, with hypercharge $Y = 1/3$. The superscript I implies that the fermion fields are expressed in the *interaction (flavour)* basis. The subscript i stands for the three generations (families). Explicitly, $Q^I_{Li}(3,2,+1/3)$ is therefor a shorthand notation for:

$$Q^I_{Li}(3,2,+1/3) = \begin{pmatrix} u^I_g, u^I_r, u^I_b \\ d^I_g, d^I_r, d^I_b \end{pmatrix}_i = \begin{pmatrix} u^I_g, u^I_r, u^I_b \\ d^I_g, d^I_r, d^I_b \end{pmatrix}, \begin{pmatrix} c^I_g, c^I_r, c^I_b \\ s^I_g, s^I_r, s^I_b \end{pmatrix}, \begin{pmatrix} t^I_g, t^I_r, t^I_b \\ b^I_g, b^I_r, b^I_b \end{pmatrix}.$$

We saw that using the Higgs field ϕ we could construct terms in the Lagrangian of the form given in equation (13). For up and down type fermions (leaving out the hermitian conjugate term) that would allow us to write for example:

$$\begin{aligned}
\mathcal{L}_{\text{quarks}} &= -\Lambda_{\text{down}}\bar{\chi}_L\phi\psi_R - \Lambda_{\text{up}}\bar{\chi}_L\tilde{\phi}^c\psi_R \\
&= -\Lambda_{\text{down}}\frac{v}{\sqrt{2}}\ \bar{d}^I d^I - \Lambda_{\text{up}}\frac{v}{\sqrt{2}}\ \bar{u}^I u^I, \\
&= -m_d\ \bar{d}^I d^I - m_u\ \bar{u}^I u^I,
\end{aligned}$$

where the strength of the interactions between the Higgs and the fermions, the so-called Yukawa couplings, had again to be added by hand.

This looks straightforward, but there is an additional complication when you realize that in the most general realization the Λ's are matrices. This will introduce mixing between different flavours as we will see a little bit later. In the most general case, again leaving out the *h.c.*, the expression for the fermion masses is written as:

$$\begin{aligned} -\mathcal{L}_{\text{Yukawa}} &= Y_{ij}\overline{\psi_{Li}}\phi\psi_{Rj} \\ &= Y^d_{ij}\overline{Q^I_{Li}}\,\phi\, d^I_{Rj} + Y^u_{ij}\overline{Q^I_{Li}}\,\tilde{\phi}^c\, u^I_{Rj} + Y^l_{ij}\overline{L^I_{Li}}\,\phi\, l^I_{Rj}, \end{aligned} \tag{15}$$

where the last term is the mass term for the charged leptons. The matrices Y^d_{ij}, Y^u_{ij} and Y^l_{ij} are arbitrary complex matrices that connect the flavour eigenstate since also terms like Y_{uc} will appear. These terms have no easy interpretation:

$$-\mathcal{L}_{\text{Yukawa}} = \quad \ldots + \quad \underbrace{-\Lambda_{dd}\frac{v}{\sqrt{2}}\bar{u}^I u^I}_{\text{mass-term down quark}} \quad \underbrace{-\Lambda_{us}\frac{v}{\sqrt{2}}\bar{u}^I s^I}_{??} \quad \underbrace{-\Lambda_{ss}\frac{v}{\sqrt{2}}\bar{s}^I s^I}_{\text{mass-term strange quark}} \quad + \ \ldots \tag{16}$$

To interpret the fields in the theory as physical particles, the fields in our model should have a well-defined mass. This is not the case in equation (16). If we write out all Yukawa terms in the Lagrangian we realize that it is possible to re-write them in terms of mixed fields that *do* have a well-defined mass. These states are the physical particles in the theory

Writing out the full Yukawa terms:

Since this is the crucial part of flavour physics, we spell out the term $Y^d_{ij}\overline{Q^I_{Li}}\,\phi\, d^I_{Rj}$ explicitly and forget about the other 2 terms in expression (15):

$$Y^d_{ij}\overline{Q^I_{Li}}\,\phi\, d^I_{Rj} \quad = \quad Y^d_{ij}\overline{(\text{up-type}\ \ \text{down-type})^I_{iL}}\begin{pmatrix}\phi^+\\ \phi\end{pmatrix}(\text{down-type})^I_{Rj} =$$

$$\begin{pmatrix} Y_{11}\overline{(u\ \ d)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} & Y_{12}\overline{(u\ \ d)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} & Y_{13}\overline{(u\ \ d)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \\ Y_{21}\overline{(c\ \ s)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} & Y_{22}\overline{(c\ \ s)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} & Y_{23}\overline{(c\ \ s)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \\ Y_{31}\overline{(t\ \ b)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} & Y_{32}\overline{(t\ \ b)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} & Y_{33}\overline{(t\ \ b)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \end{pmatrix} \cdot \begin{pmatrix} d^I_R\\ s^I_R\\ b^I_R\end{pmatrix}$$

After symmetry breaking we get the following mass terms for the fermion fields:

$$\begin{aligned} -\mathcal{L}^{\text{quarks}}_{\text{Yukawa}} &= Y^d_{ij}\overline{Q^I_{Li}}\,\phi\, d^I_{Rj} + Y^u_{ij}\overline{Q^I_{Li}}\,\tilde{\phi}\, u^I_{Rj} \\ &= Y^d_{ij}\overline{d^I_{Li}}\,\frac{v}{\sqrt{2}}\, d^I_{Rj} + Y^u_{ij}\overline{u^I_{Li}}\,\frac{v}{\sqrt{2}}\, u^I_{Rj} + ... \\ &= M^d_{ij}\overline{d^I_{Li}}d^I_{Rj} + M^u_{ij}\overline{u^I_{Li}}u^I_{Rj} + , \end{aligned} \tag{17}$$

where we omitted the corresponding interaction terms of the fermion fields to the Higgs field, $\bar{q}qh(x)$ and the hermitian conjugate terms. Note that the d's and u's in equation (17)

still each represent the three down-type and up-type quarks respectively, so the 'mixed'-terms are still there. To obtain mass eigenstates, i.e. states with proper mass terms, we should diagonalize the matrices M^d and M^u. We do this with unitary matrices V^d as follows:

$$M^d_{\text{diag}} = V^d_L M^d V^{d\dagger}_R$$
$$M^u_{\text{diag}} = V^u_L M^d V^{u\dagger}_R$$

Using the requirement that the matrices V are unitary ($V^{d\dagger}_L V^d_L = \mathbb{1}$) and leaving out again the hermitian conjugate terms the Lagrangian can now be expressed as follows:

$$\begin{aligned} -\mathcal{L}^{\text{quarks}}_{\text{Yukawa}} &= \overline{d^I_{Li}}\ M^d_{ij}\ d^I_{Rj} + \overline{u^I_{Li}}\ M^u_{ij}\ u^I_{Rj} + ... \\ &= \overline{d^I_{Li}}\ V^{d\dagger}_L V^d_L M^d_{ij} V^{d\dagger}_R V^d_R\ d^I_{Rj} + \overline{u^I_{Li}}\ V^{u\dagger}_L V^u_L M^u_{ij} V^{u\dagger}_R V^u_R\ u^I_{Rj} + ... \\ &= \overline{d_{Li}}\ (M^d_{ij})_{\text{diag}}\ d_{Rj} + \overline{u_{Li}}\ (M^u_{ij})_{\text{diag}}\ u_{Rj} + ..., \end{aligned}$$

where in the last line the matrices V have been absorbed in the quark states. Note that the up-type and down-type fields are now no longer the interaction states u^I and d^I, but are now 'simply' u and d. A bit more explicit, we now have the following quark mass eigenstates:

$$\begin{aligned} d_{Li} &= (V^d_L)_{ij} d^I_{Lj} \quad & d_{Ri} &= (V^d_R)_{ij} d^I_{Rj} \\ u_{Li} &= (V^u_L)_{ij} u^I_{Lj} \quad & u_{Ri} &= (V^u_R)_{ij} u^I_{Rj}, \end{aligned}$$

which allowed us to express the quark interaction eigenstates $d^I,\ u^I$ as quark mass eigenstates $d,\ u$. It is now interesting to see how various parts of the Standard Model Langrangian change when you write them either in the mass or the interaction eigenstates.

Rewriting interaction terms using quark mass eigenstates

The interaction terms are obtained by imposing gauge invariance by replacing the partial derivative by the covariant derivate

$$\mathcal{L}_{\text{kinetic}} = i\bar{\psi}(D^\mu \gamma_\mu)\psi, \tag{18}$$

with the covariant derivative defined as $D^\mu = \partial^\mu + ig\frac{1}{2}\vec{\tau} \cdot \vec{W}_\mu$. The τ's are the Pauli matrices and W^μ_i and B^μ are the three weak interaction bosons and the single hypercharge boson, respectively. It is very natural to write the charged current interaction between the (left-handed) iso-spin doublet interaction eigenstates that are connected by W-bosons:

$$\begin{aligned} \mathcal{L}_{\text{kinetic, weak}}(Q_L) &= i\overline{Q^I_{Li}}\gamma_\mu\big(\partial^\mu + \frac{i}{2} g W^\mu_i \tau_i\big) Q^I_{Li} \\ &= i\overline{(u\ \ d)^I_{iL}}\gamma_\mu\big(\partial^\mu + \frac{i}{2} g W^\mu_i \tau_i\big) \begin{pmatrix} u \\ d \end{pmatrix}^I_{iL} \\ &= i\overline{u^I_{iL}}\gamma_\mu \partial^\mu u^I_{iL} + i\overline{d^I_{iL}}\gamma_\mu \partial^\mu d^I_{iL} - \frac{g}{\sqrt{2}}\overline{u^I_{iL}}\gamma_\mu W^{-\mu} d^I_{iL} - \frac{g}{\sqrt{2}}\overline{d^I_{iL}}\gamma_\mu W^{+\mu} u^I_{iL} + ... \end{aligned}$$

, where we used $W^{\pm} = \frac{1}{\sqrt{2}}(W_1 \mp i W_2)$, see Section 2.

If we now express the Lagrangian in terms of the quark mass eigenstates $d,\ u$ instead of the weak interaction eigenstates $d^I,\ u^I$, the 'price' to pay is that the quark mixing between families (i.e. the off-diagonal elements) appear in the charged current interaction as each of the interaction fields is now replaced by a combination of the mass eigenstates:

$$\begin{aligned}\mathcal{L}_{\text{kinetic, cc}}(Q_L) &= \frac{g}{\sqrt{2}}\overline{u^I_{iL}}\gamma_\mu W^{-\mu} d^I_{iL} + \frac{g}{\sqrt{2}}\overline{d^I_{iL}}\gamma_\mu W^{+\mu} u^I_{iL} + ... \\ &= \frac{g}{\sqrt{2}}\overline{u_{iL}}(V^u_L V^{d\dagger}_L)_{ij}\gamma_\mu W^{-\mu} d_{iL} + \frac{g}{\sqrt{2}}\overline{d_{iL}}(V^d_L V^{u\dagger}_L)_{ij}\gamma_\mu W^{+\mu} u_{iL} + ...\end{aligned}$$

The CKM matrix

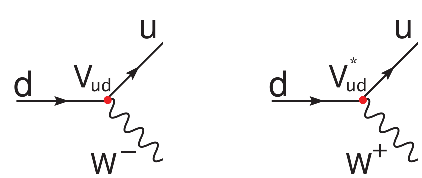
The combination of matrices $(V^d_L V^{u\dagger}_L)_{ij}$, a unitary 3×3 matrix is known under the shorthand notation V_{CKM}, the famous Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. By convention, the interaction eigenstates and the mass eigenstates are chosen to be equal for the up-type quarks, whereas the down-type quarks are chosen to be rotated, going from the interaction basis to the mass basis:

$$\begin{aligned} u^I_i &= u_j \\ d^I_i &= V_{\text{CKM}} d_j \end{aligned}$$

or explicitly:

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{19}$$

From the definition of V_{CKM} it follows that the transition from a down-type quark to an up-type quark is described by V_{ud}, whereas the transition from an up type quark to a down-type quark is described by V^*_{ud}. A separate lecture describes in detail how V_{CKM} allows for CP-violation in the SM.



Note on lepton masses

We should note here that in principle a similar matrix exists that connects the lepton *flavour* and *mass* eigenstates. In this case, contrary to the quarks, the down-type interaction doublet-states (charged leptons) are chosen to be the same as the mass eigenstates. The rotation between mass and interaction eigenstates is in the neutrino sector. This matrix is known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix and has a completely different structure than the one for quarks. Just like for the CMS matrix, the origin of the observed patterns are completely unknown. A last thing to remember: neutrino interaction eigenstates are known as $\nu_e,\ \nu_\mu$ and ν_τ, whereas the physical particles, the mass eigenstates, are $\nu_1,\ \nu_2$ and ν_3.

3.3 Higgs boson decay

It is interesting to study details of the Higgs boson properties like its coupling to fermions and gauge bosons as that determines if and how the Higgs boson is produced in experiments and what the event topology will be. In Section 3.3.3 we list all couplings and as an example we'll compute the decay rate fractions of a Higgs boson into fermions as a function of it's unknown mass in Section 3.3.1.

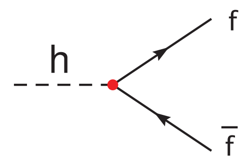
3.3.1 Higgs boson decay to fermions

Now that we have derived the coupling of fermions and gauge bosons to the Higgs field, we can look in more detail at the decay of the Higgs boson.
The general expression for the two-body decay rate:

$$\frac{d\Gamma}{d\Omega} = \frac{|\mathcal{M}|^2}{32\pi^2 s}\ |p_f|\ S, \tag{20}$$

with $\mathcal{M}$ the matrix element, $|p_f|$ the momentum of the produced particles and $S = \frac{1}{n!}$ for n identical particles. In a two-body decay we have $\sqrt{s} = m_h$ and $|p_f| = \frac{1}{2}\beta\sqrt{s}$ (see exercise 2). Since the Higgs boson is a scalar particle, the Matrix element takes a simple form:

$$\begin{aligned} -i\mathcal{M} &= \bar{u}(p_1)\frac{im_f}{v}v(-p_2) \\ i\mathcal{M}^\dagger &= \bar{v}(-p_2)\frac{-im_f}{v}u(p_1) \end{aligned}$$



Since there are no polarizations for the scalar Higgs boson, computing the Matrix element squared is 'easy':

$$\begin{aligned} \mathcal{M}^2 &= \left(\frac{m_f}{v}\right)^2 \sum_{s_1,s_2} (\bar{v})_{s_2}(-p_2)u_{s_1}(p_1)(\bar{u})_{s_1}(p_1)v_{s_2}(-p2) \\ &= \left(\frac{m_f}{v}\right)^2 \sum_{s_1} u_{s_1}(p_1)(\bar{u})_{s_1}(p_1) \sum_{s_1} \bar{v}_{s_2}(-p_2)v_{s_2}(-p_2) \\ &= \left(\frac{m_f}{v}\right)^2 \mathrm{Tr}(\not{p}_1 + m_f)\mathrm{Tr}(-\not{p}_2 - m_f) \\ &= \left(\frac{m_f}{v}\right)^2 \left[-\mathrm{Tr}(\not{p}_1 \not{p}_2) - m_f^2\mathrm{Tr}(\mathbb{1}))\right] \\ &= \left(\frac{m_f}{v}\right)^2 \left[-4p_1\cdot p_2 - 4m_f^2\right] \\ &\quad \text{use: } s = (p_1 - p_2)^2 = p_1^2 + p_2^2 - 2p_1\cdot p_2 \text{ and since } p_1^2 = p_2^2 = m_f^2 \\ &\quad\quad \text{and } s = m_h^2 \text{ we have } m_h^2 = 2m_f^2 - 2p_1\cdot p_2 \\ &= \left(\frac{m_f}{v}\right)^2 \left[2m_h^2 - 8m_f^2\right] \\ &= \left(\frac{m_f}{v}\right)^2 2m_h^2\beta^2 \qquad , \text{with } \beta = \sqrt{1 - \frac{4m_f^2}{m_h^2}} \end{aligned}$$

Including the number of colours (for quarks) we finally have:

$$\mathcal{M}^2 = \left(\frac{m_f}{v}\right)^2 2m_h^2 \beta^2 N_c$$

Decay rate:

Starting from equation (20) and using $\mathcal{M}^2$ (above), $|p_f| = \frac{1}{2}\beta\sqrt{s}$, S=1 and $\sqrt{s} = m_h$ we get:

$$\frac{d\Gamma}{d\Omega} = \frac{|\mathcal{M}|^2}{32\pi^2 s}\,|p_f|\;S \quad = \quad \frac{N_c m_h}{32\pi^2}\left(\frac{m_f}{v}\right)^2 \beta^3$$

Doing the angular integration $\int d\Omega = 4\pi$ we finally end up with:

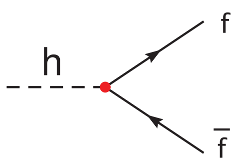
$$\Gamma(h \to f\bar{f}) = \frac{N_c}{8\pi v^2} m_f^2 m_h \beta_f^3.$$

3.3.2 Higgs boson decay to gauge bosons

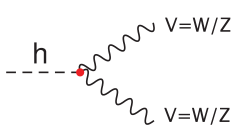
The decay ratio to gauge bosons is a bit more tricky, but is explained in great detail in Exercise 5.

3.3.3 Review Higgs boson couplings to fermions and gauge bosons

A summary of the Higgs boson couplings to fermions and gauge bosons.



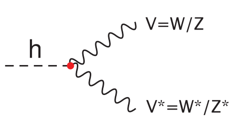
$\Gamma(h \to f\bar{f}) = \frac{N_c}{8\pi v^2}\; m_f^2\; m_h\; \sqrt{1-x}$, with $x = \frac{4m_f^2}{m_h^2}$



$\Gamma(h \to VV) = \frac{g^2}{64\pi M_W^2}\; m_h^3\; \mathcal{S}_{VV}\; (1 - x + \frac{3}{4}x^2)\sqrt{1-x}$

, with $x = \frac{4M_V^2}{m_h^2}$ and $\mathcal{S}_{WW,ZZ} = 1, \frac{1}{2}$.

The decay of the Higgs boson to two off-shell gauge bosons is given by:

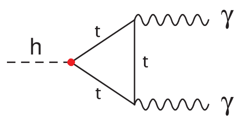


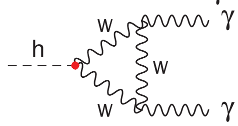
$\Gamma(h \to VV^*) = \frac{3M_V^4}{32\pi^2 v^4}\; m_h\; \delta_V'\; \mathcal{R}(x)$, with

$\delta_W' = 1,\ \delta_Z' = \frac{7}{12} - \frac{10}{9}\sin^2\theta_W + \frac{40}{27}\sin^4\theta_W$, with

$$\mathcal{R}(x) = \frac{3(1-8x+20x^2)}{\sqrt{4x-1}} \mathrm{acos}\left(\frac{3x-1}{2x^{3/2}}\right) - \frac{1-x}{2x}(2 - 13x + 47x^2) - \frac{3}{2}(1 - 6x + 4x^2)\ln(x)$$

Since the coupling of the Higgs boson to gauge bosons is so much larger than that to fermions, the Higgs boson decays to off-shell gauge bosons even though $M_{V^*} + M_V < 2M_V$. The increase in coupling 'wins' from the Breit-Wigner suppression. For example: at m_h= 140 GeV, the $h \to WW^*$ is already larger than $h \to b\bar{b}$.

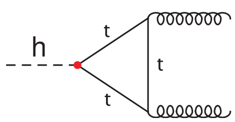




$$\Gamma(h \to \gamma\gamma) = \frac{\alpha^2}{256\pi^3 v^2}\; m_h^3 \; \left| \tfrac{4}{3} \sum_f N_c^{(f)} e_f^2 - 7 \right|^2$$

, where e_f is the fermion's electromagnetic charge.

Note: - WW contribution $\approx$ 5 times top contribution
- Some computation also gives $h \to \gamma Z$

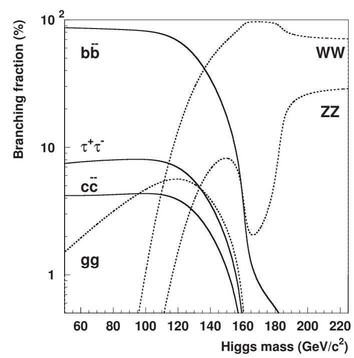


$$\Gamma(h \to \text{gluons}) = \frac{\alpha_s^2}{72\pi^3 v^2}\; m_h^3 \; \left[1 + \left(\frac{95}{4} - \frac{7N_f}{6} \right) \frac{\alpha_s}{\pi} + ... \right]^2$$

Note: - The QCD higher order terms are large.
- Reading the diagram from right to left you see the dominant production mechanism of the Higgs boson at the LHC.

3.3.4 Higgs branching fractions

Having computed the branching ratios to fermions and gauge bosons in Section 3.3.1 and Section 3.3.2 we can compute the relative branching fractions for the decay of a Higgs boson as a function of its mass. The distribution is shown here.



3.4 Theoretical bounds on the mass of the Higgs boson

Although the Higgs mass is not predicted within the minimal SM, there are theoretical upper and lower bounds on the mass of the Higgs boson if we assume there is no new physics between the electroweak scale and some higher scale called Λ. In this section we present a quick sketch of the various arguments and present the obtained limits.

3.4.1 Unitarity

In the absence of a scalar field the amplitude for elastic scattering of longitudinally polarised massive gauge bosons (e.g. $\mathrm{W_L^+W_L^-} \to \mathrm{W_L^+W_L^-}$) diverges quadratically with the centre-of-mass energy when calculated in perturbation theory and at an energy of 1.2 TeV this process violates unitarity. In the Standard Model, the Higgs boson plays an important role in the cancellation of these high-energy divergences. Once diagrams involving a scalar particle (the Higgs boson) are introduced in the gauge boson scattering mentioned above, these divergences are no longer present and the theory remains unitary and renormalizable. Focusing on solving these divergences alone also yields most of the Higgs bosons properties. This cancellation only works however if the Higgs boson is not too heavy. By requiring

that perturbation theory remains valid an upper limit on the Higgs mass can be extracted. With the requirement of unitarity and using all (coupled) gauge boson scattering processes it can be shown that:

$$m_h \ < \ \sqrt{\frac{4\pi\sqrt{2}}{3G_F}} \ \sim 700 \ \ \text{GeV/c}^2.$$

It is important to note that this does not mean that the Higgs boson can not be heavier than 700 GeV/c^2. It only means that for heavier Higgs masses, perturbation theory is not valid and the theory is not renormalisable.

This number comes from an analysis that uses a partial wave decomposition for the matrix element $\mathcal{M}$, i.e.:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi s}\mathcal{M}^2 \quad \text{, with} \quad \mathcal{M} = 16\pi\sum_{l=0}^{l=\infty}(2l+1)\mathrm{P}_l(\cos\theta)a_l,$$

where P_l are Legendre polynomials and a_l are spin-l partial waves. Since $(W_L^+W_L^- + Z_L + Z_L + HH)^2$ is well behaved, it must respect unitarity, i.e. $|a_i| < 1$ or $|Re(a_i)| \leq 0.5$. As the largest amplitude is given by:

$$a_0^{\max} = -\frac{G_F m_h^2}{4\pi\sqrt{2}} \ \cdot \ \frac{3}{2}$$

This can then be transformed into an upper limit on m_h:

$$\begin{aligned} |a_0| < \frac{1}{2} \rightarrow m_h^2 \ &< \ \frac{8\pi\sqrt{2}}{6G_F} \ \left(= \ \frac{8}{3}\pi v^2 \text{ using } G_F = \frac{1}{\sqrt{2}v^2}\right) \\ m_h \ &< \ 700 \text{ GeV} \quad \text{using } v\text{= 246 GeV.} \end{aligned}$$

This limit is soft, i.e. it means that for Higgs boson masses > 700 GeV perturbation theory breaks down.

3.4.2 Triviality and Vacuum stability

In this section, the running of the Higgs self-coupling λ with the renormalisation scale μ is used to put both a theoretical upper and a lower limit on the mass of the Higgs boson as a function of the energy scale Λ.

Running Higgs coupling constant

Similar to the gauge coupling constants, the coupling λ 'runs' with energy.

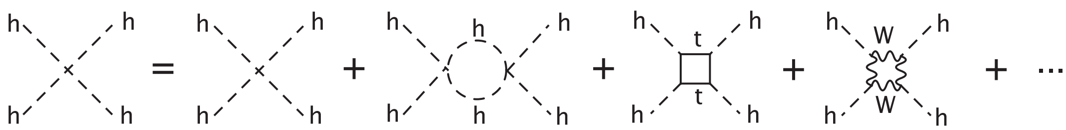
$$\frac{d\lambda}{dt} = \beta_\lambda \qquad \text{, where } t = \ln(Q^2).$$

Although these evolution functions (called β-functions) have been calculated for all SM couplings up to two loops, to focus on the physics, we sketch the arguments to obtain these

mass limits by using only the one-loop results. At one-loop the quartic coupling runs with the renormalisation scale as:

$$\frac{d\lambda}{dt} \;\equiv\; \beta_\lambda \;=\; \frac{3}{4\pi^2}\left[\lambda^2 + \frac{1}{2}\lambda h_t^2 - \frac{1}{4}h_t^4 + \mathcal{B}(g, g')\right] \tag{21}$$

, where h_t is the top-Higgs Yukawa coupling as given in equation (13). The dominant terms in the expression are the terms involving the Higgs self-coupling λ and the top quark Yukawa coupling h_t. The contribution from the gauge bosons is small and explicitly given by $\mathcal{B}(g, g') = -\frac{1}{8}\lambda(3g^2 + g'^2) + \frac{1}{64}(3g^4 + 2g^2g'^2 + g'^4)$. The terms involving the mass of the Higgs boson, top quark and gauge bosons can be understood from looking in more detail at the effective coupling at higher energy scales, where contributions from higher order diagrams enter:



This expression allows to evaluate the value of $\lambda(\Lambda)$ relative to the coupling at a reference scale which is taken to be $\lambda(v)$.

If we study the β-function in 2 special regimes: $\lambda \gg g, g', h_t$ or $\lambda \ll g, g', h_t$, we'll see that we can set both a lower *and* an upper limit on the mass of the Higgs boson as a function of the energy-scale cut-off in our theory (Λ):

$$\underbrace{\text{Triviality}}_{\substack{\text{upper bound on } m_h \\ m_h^{\max}(\Lambda)}} \qquad \text{and} \qquad \underbrace{\text{Vacuum stability}}_{\substack{\text{lower bound on } m_h \\ m_h^{\min}(\Lambda)}}$$

3.4.3 Triviality: $\lambda \gg \mathbf{g}, \mathbf{g}', \mathbf{h_t}$ heavy Higgs boson $\rightarrow$ upper limit on $\mathbf{m_h}$

For large values of λ (heavy Higgs boson since $m_h^2 = 2\lambda v^2$) and neglecting the effects from gauge interactions and the top quark, the evolution of λ is given by the dominant term in equation (21) that can be easily solved for $\lambda(\Lambda)$:

$$\frac{d\lambda}{dt} \;=\; \frac{3}{4\pi^2}\lambda^2 \qquad \Rightarrow \qquad \lambda(\Lambda) \;=\; \frac{\lambda(v)}{1 - \frac{3\lambda(v)}{4\pi^2}\ln\left(\frac{\Lambda^2}{v^2}\right)} \tag{22}$$

Note:

- We now have related λ at a scale v to λ at a higher scale Λ. We see that as Λ grows, $\lambda(\Lambda)$ grows. We should remember that $\lambda(v)$ is related to m_h: $m_h = \sqrt{-2\lambda v^2}$.
- There is a scale Λ at which $\lambda(\Lambda)$ is infinite. As Λ increases, $\lambda(\Lambda)$ increases until at $\Lambda = v\exp(2\pi^2/3\lambda(v))$ there is a singularity, known as the Landau pole.

$$\frac{3\lambda(v)}{4\pi^2}\ln\left(\frac{\Lambda^2}{v^2}\right) = 1 \rightarrow \text{At a scale } \Lambda = ve^{2\pi^2/3\lambda(v)} \quad \lambda(\Lambda) \;\text{ is infinite.}$$

If the SM is required to remain valid up to some cut-off scale Λ, i.e. if we require $\lambda(Q) < \infty$ for all $Q < \Lambda$ this puts a constraint (a maximum value) on the value of the Higgs self-coupling at the electroweak scale (v): $\lambda(v)^{\max}$ and therefore on the maximum Higgs mass since $m_h^{\max} = \sqrt{2\lambda(v)^{\max} v^2}$. Taking $\lambda(\Lambda) = \infty$ and 'evolving the coupling downwards', i.e. find $\lambda(v)$ for which $\lambda(\Lambda) = \infty$ (the Landau pole) we find:

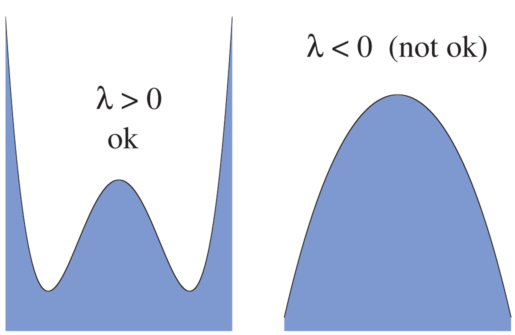
$$\lambda^{\max}(v) \;=\; \frac{4\pi^2}{3\ln\left(\frac{\Lambda^2}{v^2}\right)} \qquad \Rightarrow \quad m_h < \sqrt{\frac{8\pi^2 v^2}{3\ln\left(\frac{\Lambda^2}{v^2}\right)}} \tag{23}$$

For Λ=10^{16} GeV the upper limit on the Higgs mass is 160 GeV/c^2. This limit gets less restrictive as Λ decreases. The upper limit on the Higgs mass as a function of Λ from a computation that uses the two-loop β function and takes into account the contributions from top-quark and gauge couplings is shown in the Figure at the end of Section 3.4.4.

3.4.4 Vacuum stability $\lambda \ll \mathbf{g}, \mathbf{g}', \mathbf{h_t}$ light Higgs boson $\rightarrow$ lower limit on $\mathbf{m_h}$

For small λ (light Higgs boson since $m_h^2 = 2\lambda v^2$), a lower limit on the Higgs mass is found by the requirement that the minimum of the potential be lower than that of the unbroken theory and that the electroweak vacuum is stable. In equation (21) it is clear that for small λ the dominant contribution comes from the top quark through the Yukawa coupling ($-h_t^4$).

$$\begin{aligned}
\beta_\lambda &= \frac{1}{16\pi^2}\left[-3h_t^4 + \frac{3}{16}(2g^4 + (g^2 + g'^2)^2)\right] \\
&= \frac{3}{16\pi^2 v^4}\left[2M_W^4 + M_Z^4 - 4m_t^4\right] \\
&< 0.
\end{aligned}$$



Since this contribution is negative, there is a scale Λ for which $\lambda(\Lambda)$ becomes negative. If this happens, i.e. when $\lambda(\mu) < 0$ the potential is unbounded from below. As there is no minimum, no consistent theory can be constructed.

The requirement that λ remains positive up to a scale Λ, such that the Higgs vacuum is the global minimum below some cut-off scale, puts a lower limit on $\lambda(v)$ and therefore on the Higgs mass:

$$\frac{d\lambda}{dt} = \beta_\lambda \rightarrow \lambda(\Lambda) - \lambda(v) = \beta_\lambda \ln\left(\frac{\Lambda^2}{v^2}\right) \qquad \text{and require } \lambda(\Lambda) > 0.$$

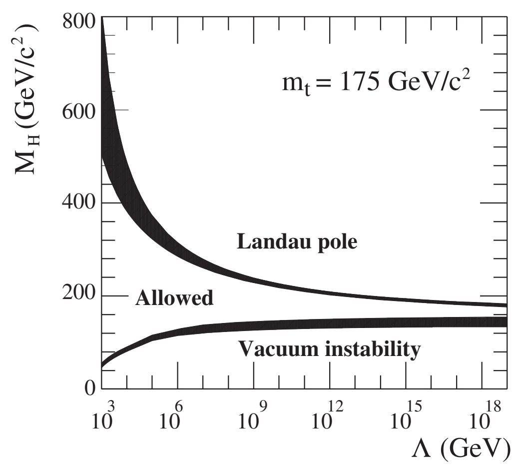
$$\begin{aligned}
\lambda(v) &> \beta_\lambda \ln\left(\frac{\Lambda^2}{v^2}\right) \text{ and } \lambda^{\min}(v) \to (m_h^{\min})^2 > 2\lambda^{\min}(v)v^2 \text{ , so} \\
m_h^2 &> 2v^2\beta_\lambda \ln\left(\frac{\Lambda^2}{v^2}\right) \\
(m_h^{\min})^2 &= \frac{3}{8\pi^2 v^2}\left[2M_W^4 + M_Z^4 - 4m_t^4\right] \\
&> -493 \ln\left(\frac{\Lambda^2}{v^2}\right)
\end{aligned}$$

Note: This result makes no sense, but is meant to describe the logic. If we go to the 2-loop beta-function we get a new limit: $m_h > 130 - 140$ GeV if $\Lambda = 10^{19}$ GeV. A detailed evaluation taking into account these considerations has been performed. The region of excluded Higgs masses as a function of the scale Λ from this analysis is also shown in the Figure at the end of Section 3.4.4 by the lower excluded region.

Summary of the theoretical bounds on the Higgs mass

In the Figure on the right the theoretically allowed range of Higgs masses is shown as a function of Λ.

For a small window of Higgs masses around 160 GeV/c^2 the Standard Model is valid up to the Planck scale ($\sim 10^{19}$ GeV). For other values of the Higgs mass the Standard Model is only an effective theory at low energy and new physics has to set in at some scale Λ.



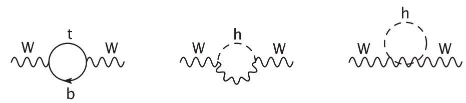
3.5 Experimental limits on the mass of the Higgs boson

3.5.1 Indirect measurements

The electroweak gauge sector of the SM is described by only three independent parameters: g, g' and v. The predictions for electroweak observables, are often presented using three (related) variables that are known to high precision: G_F, M_Z and α_{QED}. To obtain predictions to a precision better than the experimental uncertainties (often at the per mill level) higher order loop corrections have to be computed. These higher order radiative corrections contain, among others, contributions from the mass of the top quark and the Higgs boson. Via the precision measurements one is sensitive to these small contributions and thereby to the masses of these particles.

Radiative corrections

An illustration of the possibility to estimate the mass of a heavy particle entering loop corrections is the very good agreement between the estimate of the top quark mass using only indirect measurements and the direct observation.



$$
\begin{aligned}
\text{Estimate:} \quad & m_t &= 177.2^{+2.9}_{-3.1}\ \text{GeV/c}^2 \\
\text{Measurement:} \quad & m_t &= 173.2 \pm 0.9\ \text{GeV/c}^2
\end{aligned}
$$

Sensitivity to Higgs boson mass through loop corrections

Apart from the mass of the W-boson, there are more measurements that provide sensitivity to the mass of the Higgs boson. A summary of the measurements of several SM measurements is given in the left plot of Figures 1.
While the corrections connected to the top quark behave as m_t^2, the sensitivity to the mass of the Higgs boson is unfortunately only logarithmic ($\sim \ln m_h$):

$$
\begin{aligned}
\rho &= \frac{\mathrm{M_W^2}}{\mathrm{M_Z^2 cos}\theta_\mathrm{W}}\left[1 + \Delta_\rho^{\text{quarks}} + \Delta_\rho^{\text{higgs}} + \ldots\right] \\
&= \frac{\mathrm{M_W^2}}{\mathrm{M_Z^2}\cos\theta_\mathrm{W}}\left[1 + \frac{3}{16\pi^2}\left(\frac{m_t}{v}\right)^2 + 1 - \frac{11\tan\theta_W}{96\pi^2}g^2\ln\left(\frac{m_h}{\mathrm{M_W}}\right) + \ldots\right]
\end{aligned}
$$

The results from a global fit to the electroweak data with only the Higgs mass as a free parameter is shown in the right plot of Figure 1. The plot shows the $\Delta\chi^2$ distribution as a function of m_h. The green band indicates the remaining theoretical uncertainty in the fit. The result of the fit suggested a rather light Higgs boson and it could be summarised by

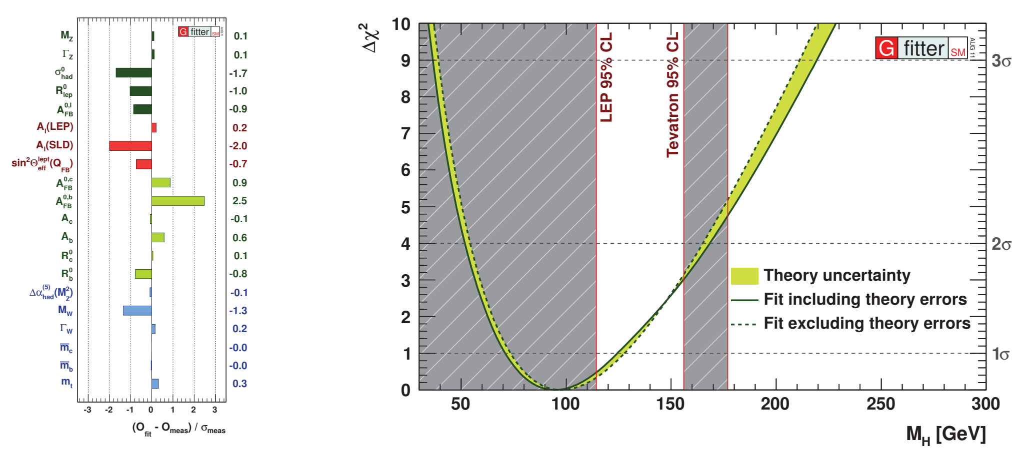


Figure 1: *Status of various SM measurements (left) and the $\Delta\chi^2$ distribution as a function of m_h from a global fit with only m_h as a free parameter (right). Before the discovery.*

the central value with its one standard deviation and the one-sided (95% CL) upper limit:

$$
m_\mathrm{h} = 95^{+30\ +74}_{-24\ -43}\ \ \text{GeV/c}^2 \qquad \text{and} \quad m_\mathrm{h} < 162\ \ \text{GeV/c}^2 \quad \text{(at 95\% CL)}.
$$

3.5.2 Direct measurements

In July 2012 the ATLAS and CMS experiments at the Large Hadron Collider at CERN announced the discovery of the Higgs boson. We will discuss the details of the search for the Higgs boson and its discovery in a separate lecture, but we since we cannot have a lecture note on the Higgs boson without proof of its discovery I include here 4 plots that were in the discovery paper of the ATLAS experiment.

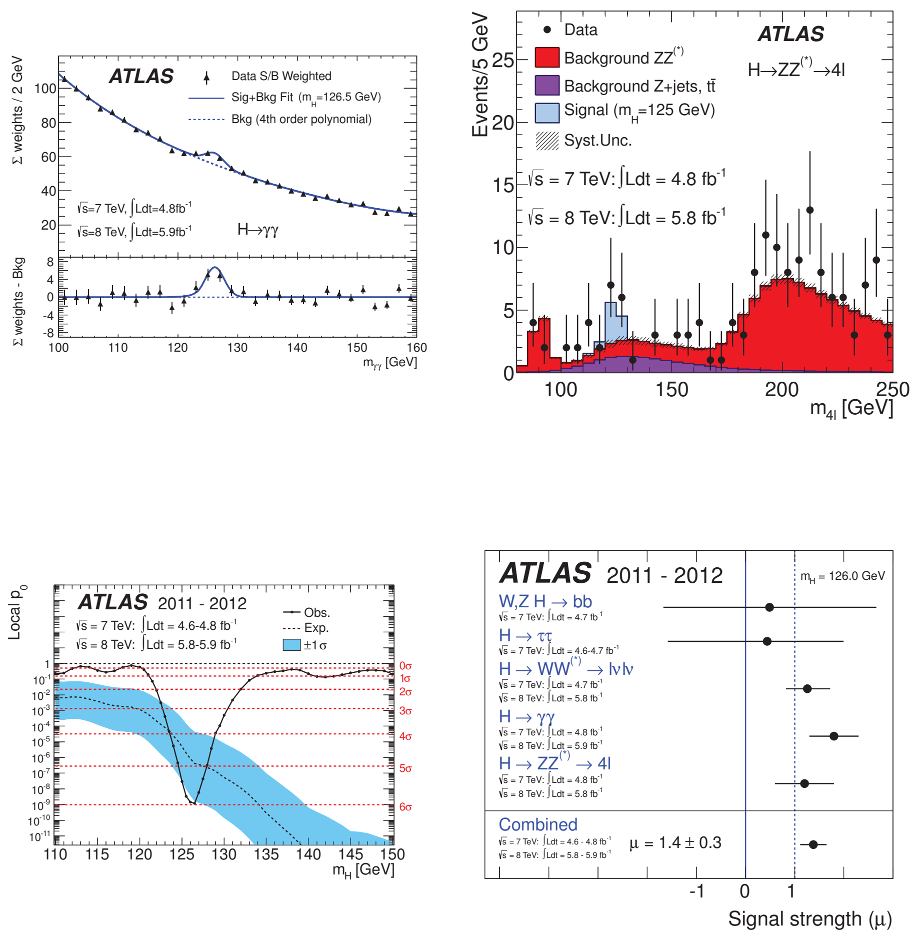


Figure 2: *Plots from the Higgs discovery paper from ATLAS. Two-photon invariant mass distribution (top left), the 4-lepton invariant mass distribution (top right), the p-value as a function of the Higgs mass (bottom left) and the measurement of the coupling strength of the Higgs boson to gauge bosons and fermions (bottom right).*

All results on the Higgs boson from the ATLAS and CMS experiments at the LHC can be found on these locations:
ATLAS: `https://twiki.cern.ch/twiki/bin/view/AtlasPublic/HiggsPublicResults`
CMS: `http://cms.web.cern.ch/org/cms-higgs-results`

Exercises lecture 3

Exercise 1): Show that $\bar{\mathrm{u}}\mathrm{u} = (\bar{\mathrm{u}}_\mathrm{L}\mathrm{u}_\mathrm{R} + \bar{\mathrm{u}}_\mathrm{R}\mathrm{u}_\mathrm{L})$

Exercise 2):
Show that in a two body decay (a heavy particle M decaying into two particles with mass m) the momentum of the decay particles can be written as:

$$|\mathrm{p_f}| = \frac{\sqrt{s}}{2}\beta, \text{ with } \ \beta = \sqrt{1-x} \text{ and } \ x = \frac{4m^2}{M^2}$$

Exercise 3): Higgs decay into fermions for $\mathrm{m_h} = 100$ GeV
Use $\mathrm{m_b} = 4.5$ GeV, $\mathrm{m}_\tau = 1.8$ GeV, $\mathrm{m_c} = 1.25$ GeV

a) Compute $\Gamma(\mathrm{H} \to \mathrm{b\bar{b}})$.
b) Compute $\Gamma(\mathrm{H} \to \text{all})$ assuming only decay into the three heaviest fermions.
c) What is the lifetime of the Higgs boson. Compare it to that of the Z boson.

Exercise 4) H&M exercise 6.16:
The helicity states λ of a massive vector particle can be described by polarization vectors. Show that:

$$\sum_\lambda \epsilon_\mu^{(\lambda)*}\epsilon_\nu^{(\lambda)} \quad = \quad -\,g_{\mu\nu} \ + \ \frac{p_\mu p_\nu}{M^2}$$

Exercise 5) Higgs decay to vector bosons
Computing the Higgs boson decay into gauge bosons (W/Z = V), with boson momenta p, q and helicities λ, δ is a bit more tricky. Let's go through it step by step.

a) Draw the Feynman diagram and use the vertex factor you computed last week to show that the matrix element squared is given by:

$$M^2 = \left(\frac{gM_V^2}{M_W}\right)^2 \sum_{\lambda,\delta} g_{\mu\nu}(\epsilon_\lambda^\mu)^*(\epsilon_\delta^\nu)^* g_{\alpha\beta}(\epsilon_\lambda^\alpha)(\epsilon_\delta^\beta),$$

where λ and δ are the helicity states of the Z bosons.

b) Use your results of exercise 4 and work out to show that:

$$M^2 = \left(\frac{gM_V^2}{M_W}\right)^2 \left[2 + \frac{(p\cdot q)^2}{M_V^4}\right],$$

where p and q are are the momenta of the two Z bosons.

d) Show that the matrix element can finally be written as:

$$M^2 = \frac{g^2}{4\mathrm{M_W^2}} m_h^4 (1 - x + \frac{3}{4}x^2), \text{ with } \mathrm{x} = \frac{4\mathrm{M_V^2}}{m_h^2}$$

e) Show that the Higgs decay into vector bosons can be written as:

$$\Gamma(h \to VV) = \frac{g^2 S_{VV}}{64\pi \mathrm{M}_{\mathrm{W}}^2} m_h^3 (1 - x + \frac{3}{4}x^2)\sqrt{1-x},$$

with x $= \frac{4M_V^2}{m_h^2}$ and $S_{\mathrm{WW,ZZ}} = 1, \frac{1}{2}$.

f) Compute Γ(h$\to$ WW) for $m_h = 200$ GeV.
What is the total width (only WW and ZZ decays)? And the lifetime ?

4 Problems with the Higgs mechanism and Higgs searches

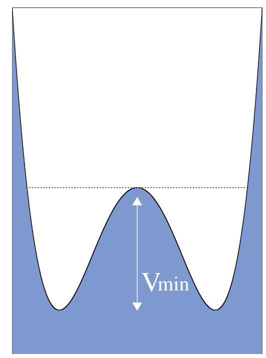
Although the Higgs mechanism cures many of the problems in the Standard Model, there are also several 'problems' associated to the Higgs mechanism. We will explore these problems in this section and very briefly discuss the properties of non-SM Higgs bosons.

4.1 Problems with the Higgs boson

4.1.1 Problems with the Higgs boson: Higgs self-energy

Since the Higgs field occupies all of space, the non-zero vacuum expectation value of the Higgs field (v) will contribute to the vacuum energy, i.e. it will contribute to the cosmological constant in Einstein's equations: $\Lambda = \frac{8\pi G_N}{c^4}\rho_{\rm vac}$.

Energy density Higgs field:
With $V(\phi^\dagger\phi) = \mu^2\phi^2 + \lambda\phi^4$, The 'depth' of the potential is:



$$
\begin{aligned}
V_{\rm min} = V(v) &= \frac{1}{2}\mu^2 v^2 + \frac{1}{4}\lambda v^4 \quad \text{use } \mu^2 = -\lambda v^2 \\
&= -\frac{1}{4}\lambda v^4 \qquad\qquad \text{use } m_h^2 = 2\lambda v^2 \\
&= -\frac{1}{8} m_h^2 v^2
\end{aligned}
$$

Note that we cannot simply redefine $\rm V_{min}$ to be 0, or any arbitrary number since quantum corrections will always yield a value like the one (order of magnitude) given above. The Higgs mass is unknown, but since we have a lower limit on the (Standard Model) Higgs boson mass from direct searches at LEP ($m_h > 114.4$ GeV/c^2) we *can* compute the contribution of the Higgs field to $\rho_{\rm vac}$.

$$
\begin{aligned}
\rho_{\rm vac}^{\rm Higgs} &= \frac{1}{8} m_h^2 v^2 \\
&> 1 \cdot 10^8 \ {\rm GeV}^4 \qquad \text{and since GeV} = \frac{1}{r} \\
&> 1 \cdot 10^8 \ {\rm GeV/r}^3 \qquad \text{(energy density)}
\end{aligned}
$$

Measured vacuum energy density:
An experiment to measure the energy density in vacuum and the energy density in matter has shown:

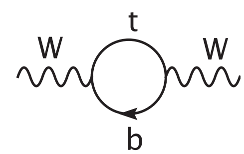
$$\Omega_m \approx 30\% \ \text{ and } \ \Omega_\Lambda \approx 70\% \sim 10^{-46}{\rm GeV}^4 \quad \rightarrow \quad \text{empty space is really quite empty.}$$

Problem:
- 10^{54} orders of magnitude mismatch.
- Why is the universe larger than a football ?

4.1.2 Problems with the Higgs boson: the hierarchy problem

In the electroweak theory of the SM, loop corrections are small. In the loops the integration is done over momenta up to a cut-off value Λ.

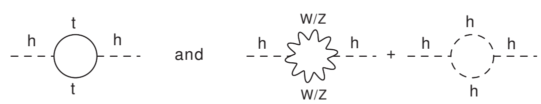
Success of radiative corrections:
When we discussed the sensitivity of the electroweak measurements to the mass of the Higgs boson through the radiative corrections, the example of the prediction of the top quark mass was mentioned:



Indirect estimate:	$m_t = 178^{+9.8}_{-4.2}$ GeV/c^2
Direct result:	$m_t = 172.4 \pm 1.2$ GeV/c^2

Failure of radiative corrections:
Also the Higgs propagator receives quantum corrections.

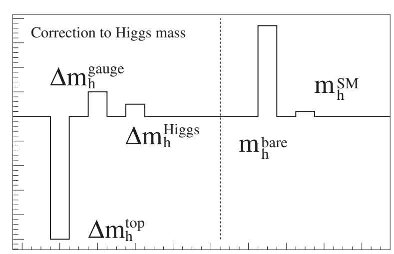
$$m_h = m_h^{\text{bare}} + \Delta m_h^{\text{ferm.}} + \Delta m_h^{\text{gauge}} + \Delta m_h^{\text{Higgs}} + ...$$



The corrections from the fermions (mainly from the top quark) are large. Expressed in terms of the loop-momentum cut-off Λ given by:

$$\left(\Delta m_h^2\right)^{\text{top}} = -\frac{3}{8\pi^2}\lambda_t^2\Lambda^2$$

The corrections from the top quark are not small at all, but huge and of order Λ. If Λ is chosen as 10^{16} (GUT) or 10^{19} (Planck), and taking the corrections into account (same order of magnitude), it is unnatural for m_h to be of order of $\mathrm{M_{EW}}(\approx v)$.



The hierarchy problem: why is $\mathrm{M_{EW}} \ll \mathrm{M_{PL}}$?

Most popular theoretical solution to the hierarchy problem is the concept of Supersymmetry, where for every fermion/boson there is a boson/fermion as partner. For example, the top and stop (supersymmetric bosonic partner of the top quark) contributions (almost) cancel. The quadratic divergences have disappeared and we are left with

$$\Delta m_h^2 \propto (m_f^2 - m_S^2)\ln\left(\frac{\Lambda}{m_S}\right).$$

4.2 Higgs bosons in models beyond the SM (SUSY)

When moving to a supersymmetric description of nature we can no longer use a single Higgs doublet, but will need to introduce at least two, because:

A) In the SM we used $\phi/\tilde{\phi}^c$ to give mass to down/up-type particles in $\mathrm{SU(2)_L}$ doublets. In susy models these two terms cannot appear together in the Lagrangian. We need

an additional Higgs doublet to give mass to the up-type particles.

B) Anomalies disappear only if in a loop $\sum_f \mathrm{Y}_f = 0$. In SUSY there is an additional fermion in the model: the partner for the Higgs boson, the Higgsino. This will introduce an anomaly unless there is a second Higgsino with opposite hypercharge.

$$\phi_1 = \underbrace{\begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \to \begin{pmatrix} 0 \\ v_1 \end{pmatrix}}_{\mathrm{Y}_{\phi_1}=+1} \quad \text{and} \quad \phi_2 = \underbrace{\begin{pmatrix} \phi_2^0 \\ \phi_2^- \end{pmatrix} \to \begin{pmatrix} v_2 \\ 0 \end{pmatrix}}_{\mathrm{Y}_{\phi_2}=-1}$$

Number of degrees of freedom in SUSY models:

SM: Add 4 degrees of freedom $\to$ 3 massive gauge bosons $\to$ 1 Higgs boson (h)
SUSY: Add 8 degrees of freedom $\to$ 3 massive gauge bosons $\to$ 5 Higgs boson (h, H, A, H^+, H^-)

parameters: $\tan(\beta) = \frac{v_2}{v_1}$ and $\mathrm{M_A}$.

Note: - Sometimes people choose α = mixing angle to give h,A, similar to W_3/B_μ-mixing to give Z-boson and photon.
- $\mathrm{M_W} = \frac{1}{2}\sqrt{\mathrm{v}_1^2 + \mathrm{v}_2^2}\ \mathrm{g} \to \mathrm{v}_1^2 + \mathrm{v}_2^2 = \mathrm{v}^2$ (246 GeV).

Differences SM and SUSY Higgses:

With the new parameters, all couplings to gauge bosons and fermions change:

$$\begin{aligned}
g_{hVV}^{\mathrm{SUSY}} &= g_{hVV}^{\mathrm{SM}} \ \sin(\beta-\alpha) \\
g_{hb\bar{b}}^{\mathrm{SUSY}} &= g_{hb\bar{b}}^{\mathrm{SM}} \ -\frac{\sin\alpha}{\cos\beta} \to \frac{\Gamma(h \to b\bar{b})^{\mathrm{SUSY}}}{\Gamma(h \to b\bar{b})^{\mathrm{SM}}} = \frac{\sin^2(\alpha)}{\cos^2(\beta)} \\
g_{ht\bar{t}}^{\mathrm{SUSY}} &= g_{ht\bar{t}}^{\mathrm{SM}} \ -\frac{\cos\alpha}{\sin\beta} \to \frac{\Gamma(h \to t\bar{t})^{\mathrm{SUSY}}}{\Gamma(h \to t\bar{t})^{\mathrm{SM}}} = \frac{\cos^2(\alpha)}{\sin^2(\beta)}
\end{aligned}$$

To determine if an observed Higgs sparticle is a SM or SUSY Higgs a detailed investigation of the branching fraction is required. Unfortunately, also SUSY does not give a prediction for the lightest Higgs boson mass:

$$\begin{aligned}
m_h^2 &< M_Z^2 + \delta^2 m_{top} + \delta^2 m_X + ... \\
&\leq 130 \text{ GeV}.
\end{aligned}$$

Exercises lecture 4

Exercise 1): b-tagging at LEP.
A Higgs boson of 100 GeV decays at LEP: given a lifetime of a B mesons of roughly 1.6 picoseconds, what distance does it travel in the detector before decaying ? What is the most likely decay distance ?

Exercise 2): H $\rightarrow$ ZZ $\rightarrow$ 4 leptons at the LHC (lepton = e/μ).

a) Why is there a 'dip' in te fraction of Higgs bosons that decays to 2 Z bosons (between 160 and 180 GeV)?

b) How many events H $\rightarrow$ ZZ $\rightarrow e^+e^-\mu^+\mu^-$ muons are produced in 1 fb^{-1} of data for m_h= 140, 160, 180 and 200 GeV ? The expected number of evets is the product of the luminosity and the cross-section: $N = \mathcal{L} \cdot \sigma$

On the LHC slides, one of the LHC experiments shows its expectation for an analysis aimed at trying to find the Higgs boson in the channel with 2 electrons and 2 muons. We concentrate on m_h=140 GeV.

c) What is the fraction of events in which all 4 leptons have been well reconstructed in the detector ? What is the single (high-energy) lepton detection efficiency ? Name reasons why not all leptons are detected.

We do a counting experiment using the two bins around the expected Higgs boson mass (we assume for the moment that the background is extremely well known and does not fluctuate). In a counting experiment a Poisson distribution describes the probabilities to observe x events when λ are expected:

$$P(x|\lambda) = \frac{\lambda^x \, e^{-\lambda}}{x!}$$

d) Does this experiment expect to be able to discover the m_h=140 GeV hypothesis after 9.3 fb^{-1}.

e) Imagine the data points was the actual measurement after 9.3 fb^{-1}. Can this experiment claim to have discovered the Higgs boson at m_h=140 GeV?

Article Englert & Brout (Augustus 1964)

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BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

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It is of interest to inquire whether gauge vector mesons acquire mass through interaction[1]; by a gauge vector meson we mean a Yang-Mills field[2] associated with the extension of a Lie group from global to local symmetry. The importance of this problem resides in the possibility that strong-interaction physics originates from massive gauge fields related to a system of conserved currents.[3] In this note, we shall show that in certain cases vector mesons do indeed acquire mass when the vacuum is degenerate with respect to a compact Lie group.

Theories with degenerate vacuum (broken symmetry) have been the subject of intensive study since their inception by Nambu.[4-6] A characteristic feature of such theories is the possible existence of zero-mass bosons which tend to restore the symmetry.[7,8] We shall show that it is precisely these singularities which maintain the gauge invariance of the theory, despite the fact that the vector meson acquires mass.

We shall first treat the case where the original fields are a set of bosons φ_A which transform as a basis for a representation of a compact Lie group. This example should be considered as a rather general phenomenological model. As such, we shall not study the particular mechanism by which the symmetry is broken but simply assume that such a mechanism exists. A calculation performed in lowest order perturbation theory indicates that those vector mesons which are coupled to currents that "rotate" the original vacuum are the ones which acquire mass [see Eq. (6)].

We shall then examine a particular model based on chirality invariance which may have a more fundamental significance. Here we begin with a chirality-invariant Lagrangian and introduce both vector and pseudovector gauge fields, thereby guaranteeing invariance under both local phase and local γ_5-phase transformations. In this model the gauge fields themselves may break the γ_5 invariance leading to a mass for the original Fermi field. We shall show in this case that the pseudovector field acquires mass.

In the last paragraph we sketch a simple argument which renders these results reasonable.

(1) Lest the simplicity of the argument be shrouded in a cloud of indices, we first consider a one-parameter Abelian group, representing, for example, the phase transformation of a charged boson; we then present the generalization to an arbitrary compact Lie group.

The interaction between the φ and the A_μ fields is

$$H_{\text{int}} = ieA_\mu \varphi^* \overleftrightarrow{\partial}_\mu \varphi - e^2 \varphi^* \varphi A_\mu A_\mu, \tag{1}$$

where $\varphi = (\varphi_1 + i\varphi_2)/\sqrt{2}$. We shall break the symmetry by fixing $\langle\varphi\rangle \neq 0$ in the vacuum, with the phase chosen for convenience such that $\langle\varphi\rangle = \langle\varphi^*\rangle = \langle\varphi_1\rangle/\sqrt{2}$.

We shall assume that the application of the

theorem of Goldstone, Salam, and Weinberg[7] is straightforward and thus that the propagator of the field φ_2, which is "orthogonal" to φ_1, has a pole at $q=0$ which is not isolated.

We calculate the vacuum polarization loop $\Pi_{\mu\nu}$ for the field A_μ in lowest order perturbation theory about the self-consistent vacuum. We take into consideration only the broken-symmetry diagrams (Fig. 1). The conventional terms do not lead to a mass in this approximation if gauge invariance is carefully maintained. One evaluates directly

$$\Pi_{\mu\nu}(q)=(2\pi)^4 ie^2[g_{\mu\nu}\langle\varphi_1\rangle^2-(q_\mu q_\nu/q^2)\langle\varphi_1\rangle^2]. \quad (2)$$

Here we have used for the propagator of φ_2 the value $[i/(2\pi)^4]/q^2$; the fact that the renormalization constant is 1 is consistent with our approximation.[9] We then note that Eq. (2) both maintains gauge invariance ($\Pi_{\mu\nu}q_\nu=0$) and causes the A_μ field to acquire a mass

$$\mu^2=e^2\langle\varphi_1\rangle^2. \quad (3)$$

We have not yet constructed a proof in arbitrary order; however, the similar appearance of higher order graphs leads one to surmise the general truth of the theorem.

Consider now, in general, a set of boson-field operators φ_A (which we may always choose to be Hermitian) and the associated Yang-Mills field $A_{a,\mu}$. The Lagrangian is invariant under the transformation[10]

$$\delta\varphi_A=\textstyle\sum_{a,A}\epsilon_a(x)T_{a,AB}\varphi_B,$$

$$\delta A_{a,\mu}=\textstyle\sum_{c,b}\epsilon_c(x)c_{acb}A_{b,\mu}+\partial_\mu\epsilon_a(x), \quad (4)$$

where c_{abc} are the structure constants of a compact Lie group and $T_{a,AB}$ the antisymmetric generators of the group in the representation defined by the φ_B.

Suppose that in the vacuum $\langle\varphi_{B'}\rangle\neq 0$ for some B'. Then the propagator of $\sum_{A,B'}T_{a,AB'}\varphi_A$

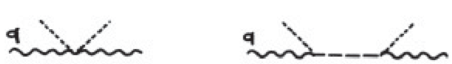

FIG. 1. Broken-symmetry diagram leading to a mass for the gauge field. Short-dashed line, $\langle\varphi_1\rangle$; long-dashed line, φ_2 propagator; wavy line, A_μ propagator. (a) $\rightarrow (2\pi)^4 ie^2 g_{\mu\nu}\langle\varphi_1\rangle^2$, (b) $\rightarrow -(2\pi)^4 ie^2(q_\mu q_\nu/q^2)\times\langle\varphi_1\rangle^2$.

$\times\langle\varphi_{B'}\rangle$ is, in the lowest order,

$$\left[\frac{i}{(2\pi)^4}\right]\sum_{A,B',C'}\frac{T_{a,AB'}\langle\varphi_{B'}\rangle T_{a,AC'}\langle\varphi_{C'}\rangle}{q^2}$$

$$\equiv\left[\frac{-i}{(2\pi)^4}\right]\frac{(\langle\varphi\rangle T_a T_a\langle\varphi\rangle)}{q^2}.$$

With λ the coupling constant of the Yang-Mills field, the same calculation as before yields

$$\Pi_{\mu\nu}{}^a(q)=-i(2\pi)^4\lambda^2(\langle\varphi\rangle T_a T_a\langle\varphi\rangle)$$

$$\times[g_{\mu\nu}-q_\mu q_\nu/q^2],$$

giving a value for the mass

$$\mu_a{}^2=-(\langle\varphi\rangle T_a T_a\langle\varphi\rangle). \quad (6)$$

(2) Consider the interaction Hamiltonian

$$H_{\rm int}=-\eta\bar\psi\gamma_\mu\gamma_5\psi B_\mu-\epsilon\bar\psi\gamma_\mu\psi A_\mu, \quad (7)$$

where A_μ and B_μ are vector and pseudovector gauge fields. The vector field causes attraction whereas the pseudovector leads to repulsion between particle and antiparticle. For a suitable choice of ϵ and η there exists, as in Johnson's model,[11] a broken-symmetry solution corresponding to an arbitrary mass m for the ψ field fixing the scale of the problem. Thus the fermion propagator $S(p)$ is

$$S^{-1}(p)=\gamma p-\Sigma(p)=\gamma p[1-\Sigma_2(p^2)]-\Sigma_1(p^2), \quad (8)$$

with

$$\Sigma_1(p^2)\neq 0$$

and

$$m[1-\Sigma_2(m^2)]-\Sigma_1(m^2)=0.$$

We define the gauage-invariant current $J_\mu{}^5$ by using Johnson's method[12]:

$$J_\mu{}^5=-\eta\lim_{\xi\to 0}\bar\psi'(x+\xi)\gamma_\mu\gamma_5\psi'(x),$$

$$\psi'(x)=\exp[-i\int_{-\infty}^x\eta B_\mu(y)dy^\mu\gamma_5]\psi(x). \quad (9)$$

This gives for the polarization tensor of the

pseudovector field

$$\Pi_{\mu\nu}{}^5(q) = \eta^2 \frac{i}{(2\pi)^4} \int \mathrm{Tr}\{S(p-\tfrac{1}{2}q)\Gamma_{\nu 5}(p-\tfrac{1}{2}q; p+\tfrac{1}{2}q) \times S(p+\tfrac{1}{2}q)\gamma_\mu\gamma_5 - S(p)[\partial S^{-1}(p)/\partial p_\nu]S(p)\gamma_\mu\} d^4p, \qquad (10)$$

where the vertex function $\Gamma_{\nu 5} = \gamma_\nu\gamma_5 + \Lambda_{\nu 5}$ satisfies the Ward identity[5]

$$q_\nu \Lambda_{\nu 5}(p-\tfrac{1}{2}q; p+\tfrac{1}{2}q) = \Sigma(p-\tfrac{1}{2}q)\gamma_5 + \gamma_5\Sigma(p+\tfrac{1}{2}q), \qquad (11)$$

which for low q reads

$$q_\nu\Gamma_{\nu 5} = q_\nu\gamma_\nu\gamma_5[1-\Sigma_2] + 2\Sigma_1\gamma_5 - 2(q_\nu p_\nu)(\gamma_\lambda p_\lambda)(\partial\Sigma_2/\partial p^2)\gamma_5. \qquad (12)$$

The singularity in the longitudinal $\Gamma_{\nu 5}$ vertex due to the broken-symmetry term $2\Sigma_1\gamma_5$ in the Ward identity leads to a nonvanishing gauge-invariant $\Pi_{\mu\nu}{}^5(q)$ in the limit $q \to 0$, while the usual spurious "photon mass" drops because of the second term in (10). The mass of the pseudovector field is roughly $\eta^2 m^2$ as can be checked by inserting into (10) the lowest approximation for $\Gamma_{\nu 5}$ consistant with the Ward identity.

Thus, in this case the general feature of the phenomenological boson system survives. We would like to emphasize that here the symmetry is broken through the gauge fields themselves. One might hope that such a feature is quite general and is possibly instrumental in the realization of Sakurai's program.[3]

(3) We present below a simple argument which indicates why the gauge vector field need not have zero mass in the presence of broken symmetry. Let us recall that these fields were introduced in the first place in order to extend the symmetry group to transformations which were different at various space-time points. Thus one expects that when the group transformations become homogeneous in space-time, that is $q \to 0$, no dynamical manifestation of these fields should appear. This means that it should cost no energy to create a Yang-Mills quantum at $q = 0$ and thus the mass is zero. However, if we break gauge invariance of the first kind and still maintain gauge invariance of the second kind this reasoning is obviously incorrect. Indeed, in Fig. 1, one sees that the A_μ propagator connects to intermediate states, which are "rotated" vacua. This is seen most clearly by writing $\langle\varphi_1\rangle = \langle[Q\varphi_2]\rangle$ where Q is the group generator. This effect cannot vanish in the limit $q \to 0$.

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Article Higgs (September 1964)

BROKEN SYMMETRIES, MASSLESS PARTICLES AND GAUGE FIELDS

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Recently a number of people have discussed the Goldstone theorem [1,2]: that any solution of a Lorentz-invariant theory which violates an internal symmetry operation of that theory must contain a massless scalar particle. Klein and Lee [3] showed that this theorem does not necessarily apply in non-relativistic theories and implied that their considerations would apply equally well to Lorentz-invariant field theories. Gilbert [4], however, gave a proof that the failure of the Goldstone theorem in the nonrelativistic case is of a type which cannot exist when Lorentz invariance is imposed on a theory. The purpose of this note is to show that Gilbert's argument fails for an important class of field theories, that in which the conserved currents are coupled to gauge fields.

Following the procedure used by Gilbert [4], let us consider a theory of two hermitian scalar fields

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$\varphi_1(x)$, $\varphi_2(x)$ which is invariant under the phase transformation

$$\begin{aligned} \varphi_1 &\rightarrow \varphi_1 \cos\alpha + \varphi_2 \sin\alpha\,, \\ \varphi_2 &\rightarrow -\varphi_1 \sin\alpha + \varphi_2 \cos\alpha\,. \end{aligned} \tag{1}$$

Then there is a conserved current j_μ such that

$$\mathrm{i}\left[\int \mathrm{d}^3x\, j_0(x),\ \varphi_1(y)\right] = \varphi_2(y). \tag{2}$$

We assume that the Lagrangian is such that symmetry is broken by the nonvanishing of the vacuum expectation value of φ_2. Goldstone's theorem is proved by showing that the Fourier transform of $\mathrm{i}\langle[j_\mu(x),\ \varphi_1(y)]\rangle$ contains a term $2\pi\langle\varphi_2\rangle\epsilon(k_0)k_\mu\,\delta(k^2)$, where k_μ is the momentum, as a consequence of Lorentz-covariance, the conservation law and eq. (2).

Klein and Lee [3] avoided this result in the non-relativistic case by showing that the most general form of this Fourier transform is now, in Gilbert's notation,

$$\mathrm{F.T.} = k_\mu \rho_1(k^2, nk) + n_\mu \rho_2(k^2, nk) + C_3 n_\mu \delta^4(k)\,, \tag{3}$$

where n_μ, which may be taken as $(1, 0, 0, 0)$, picks out a special Lorentz frame. The conversation law then reduces eq. (3) to the less general form

$$\begin{aligned} \mathrm{F.T.} = k_\mu \delta(k^2)\rho_4(nk) + [k^2 n_\mu - k_\mu(nk)]\rho_5(k^2, nk) \\ + C_3 n_\mu \delta^4(k)\,. \end{aligned} \tag{4}$$

It turns out, on applying eq. (2), that all three terms in eq. (4) can contribute to $\langle\varphi_2\rangle$. Thus the Goldstone theorem fails if $\rho_4 = 0$, which is possible only if the other terms exist. Gilbert's remark that no special timelike vector n_μ is available in a Lorentz-covariant theory appears to rule out this possibility in such a theory.

There is however a class of relativistic field theories in which a vector n_μ does indeed play a part. This is the class of gauge theories, where an auxiliary unit timelike vector n_μ must be introduced in order to define a radiation gauge in which the vector gauge fields are well defined operators. Such theories are nevertheless Lorentz-covariant, as has been shown by Schwinger [5]. (This has, of course, long been known of the simplest such theory, quantum electrodynamics.) There seems to be no reason why the vector n_μ should not appear in the Fourier transform under consideration.

It is characteristic of gauge theories that the conservation laws hold in the strong sense, as a consequence of field equations of the form

$$\begin{aligned} j^\mu &= \partial_\nu F'^{\mu\nu}, \\ F_{\mu\nu}' &= \partial_\mu A_\nu' - \partial_\nu A_\mu'\,. \end{aligned} \tag{5}$$

Except in the case of abelian gauge theories, the fields A_μ', $F_{\mu\nu}'$ are not simply the gauge field variables A_μ, $F_{\mu\nu}$, but contain additional terms with combinations of the structure constants of the group as coefficients. Now the structure of the Fourier transform of $\mathrm{i}\langle[A_\mu'(x),\ \varphi_1(y)]\rangle$ must be given by eq. (3). Applying eq. (5) to this commutator gives us as the Fourier transform of $\mathrm{i}\langle[j_\mu(x),\ \varphi_1(y)]\rangle$ the single term $[k^2 n_\mu - k_\mu(nk)]\rho(k^2, nk)$. We have thus exorcised both Goldstone's zero-mass bosons and the "spurion" state (at $k_\mu = 0$) proposed by Klein and Lee.

In a subsequent note it will be shown, by considering some classical field theories which display broken symmetries, that the introduction of gauge fields may be expected to produce qualitative changes in the nature of the particles described by such theories after quantization.

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Article Higgs (October 1964)

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BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

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In a recent note[1] it was shown that the Goldstone theorem,[2] that Lorentz-covariant field theories in which spontaneous breakdown of symmetry under an internal Lie group occurs contain zero-mass particles, fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The purpose of the present note is to report that, as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the coupling tends to zero. This phenomenon is just the relativistic analog of the plasmon phenomenon to which Anderson[3] has drawn attention: that the scalar zero-mass excitations of a superconducting neutral Fermi gas become longitudinal plasmon modes of finite mass when the gas is charged.

The simplest theory which exhibits this behavior is a gauge-invariant version of a model used by Goldstone[2] himself: Two real[4] scalar fields φ_1, φ_2 and a real vector field A_μ interact through the Lagrangian density

$$L = -\tfrac{1}{2}(\nabla\varphi_1)^2 - \tfrac{1}{2}(\nabla\varphi_2)^2 - V(\varphi_1^2+\varphi_2^2) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad (1)$$

where

$$\nabla_\mu\varphi_1 = \partial_\mu\varphi_1 - eA_\mu\varphi_2,$$

$$\nabla_\mu\varphi_2 = \partial_\mu\varphi_2 + eA_\mu\varphi_1,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

e is a dimensionless coupling constant, and the metric is taken as $-+++$. L is invariant under simultaneous gauge transformations of the first kind on $\varphi_1 \pm i\varphi_2$ and of the second kind on A_μ. Let us suppose that $V'(\varphi_0^2)=0$, $V''(\varphi_0^2)>0$; then spontaneous breakdown of U(1) symmetry occurs. Consider the equations [derived from (1) by treating $\Delta\varphi_1$, $\Delta\varphi_2$, and A_μ as small quantities] governing the propagation of small oscillations about the "vacuum" solution $\varphi_1(x)=0$, $\varphi_2(x)=\varphi_0$:

$$\partial^\mu\{\partial_\mu(\Delta\varphi_1) - e\varphi_0 A_\mu\} = 0, \qquad (2a)$$

$$\{\partial^2 - 4\varphi_0^2 V''(\varphi_0^2)\}(\Delta\varphi_2) = 0, \qquad (2b)$$

$$\partial_\nu F^{\mu\nu} = e\varphi_0\{\partial^\mu(\Delta\varphi_1) - e\varphi_0 A^\mu\}. \qquad (2c)$$

Equation (2b) describes waves whose quanta have (bare) mass $2\varphi_0\{V''(\varphi_0^2)\}^{1/2}$; Eqs. (2a) and (2c) may be transformed, by the introduction of new variables

$$B_\mu = A_\mu - (e\varphi_0)^{-1}\partial_\mu(\Delta\varphi_1),$$
$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = F_{\mu\nu}, \qquad (3)$$

into the form

$$\partial_\mu B^\mu = 0, \quad \partial_\nu G^{\mu\nu} + e^2\varphi_0^2 B^\mu = 0. \qquad (4)$$

Equation (4) describes vector waves whose quanta have (bare) mass $e\varphi_0$. In the absence of the gauge field coupling ($e=0$) the situation is quite different: Equations (2a) and (2c) describe zero-mass scalar and vector bosons, respectively. In passing, we note that the right-hand side of (2c) is just the linear approximation to the conserved current: It is linear in the vector potential, gauge invariance being maintained by the presence of the gradient term.[5]

When one considers theoretical models in which spontaneous breakdown of symmetry under a semisimple group occurs, one encounters a variety of possible situations corresponding to the various distinct irreducible representations to which the scalar fields may belong; the gauge field always belongs to the adjoint representation.[6] The model of the most immediate interest is that in which the scalar fields form an octet under SU(3): Here one finds the possibility of two nonvanishing vacuum expectation values, which may be chosen to be the two $Y=0$, $I_3=0$ members of the octet.[7] There are two massive scalar bosons with just these quantum numbers; the remaining six components of the scalar octet combine with the corresponding components of the gauge-field octet to describe

massive vector bosons. There are two $I=\frac{1}{2}$ vector doublets, degenerate in mass between $Y=\pm1$ but with an electromagnetic mass splitting between $I_3=\pm\frac{1}{2}$, and the $I_3=\pm1$ components of a $Y=0$, $I=1$ triplet whose mass is entirely electromagnetic. The two $Y=0$, $I=0$ gauge fields remain massless: This is associated with the residual unbroken symmetry under the Abelian group generated by Y and I_3. It may be expected that when a further mechanism (presumably related to the weak interactions) is introduced in order to break Y conservation, one of these gauge fields will acquire mass, leaving the photon as the only massless vector particle. A detailed discussion of these questions will be presented elsewhere.

It is worth noting that an essential feature of the type of theory which has been described in this note is the prediction of incomplete multiplets of scalar and vector bosons.[8] It is to be

[4]In the present note the model is
classical terms; nothing is proved
theory. It should be understood, t
conclusions which are presented c
of particles are conjectures based
of linearized classical field equati
sentially the same conclusions hav
dependently by F. Englert and R. B
Letters 13, 321 (1964): These aut
same model quantum mechanically
perturbation theory about the self-

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from collective excitations of the F

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octet interacts with baryons and m
Gell-Mann–Okubo and electromagn
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scalar particles exist have been m
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IFT–P/010/2000

Standard Model: An Introduction *

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Abstract

We present a primer on the Standard Model of the electroweak interaction. Emphasis is given to the historical aspects of the theory's formulation. The radiative corrections to the Standard Model are presented and its predictions for the electroweak parameters are compared with the precise experimental data obtained at the Z pole. Finally, we make some remarks on the perspectives for the discovery of the Higgs boson, the most important challenge of the Standard Model.

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Contents

Chapter 1

Introduction

The joint description of the electromagnetic and the weak interaction by a single theory certainly is one of major achievements of the physical science in this century. The model proposed by Glashow, Salam and Weinberg in the middle sixties, has been extensively tested during the last 30 years. The discovery of neutral weak interactions and the production of intermediate vector bosons ($W^{\pm}$ and Z^0) with the expected properties increased our confidence in the model. Even after the recent precise measurements of the electroweak parameters in electron–positron collisions at the Z^0 pole, there is no experimental result that contradicts the Standard Model predictions.

The description of the electroweak interaction is implemented by a gauge theory based on the $SU(2)_L \otimes U(1)_Y$ group, which is spontaneously broken via the Higgs mechanism. The matter fields — leptons and quarks — are organized in families, with the left–handed fermions belonging to weak isodoublets while the right–handed components transform as weak isosinglets. The vector bosons, $W^{\pm}$, Z^0 and γ, that mediate the interactions are introduced via minimal coupling to the matter fields. An essential ingredient of the model is the scalar potential that is added to the Lagrangian to generate the vector–boson (and fermion) masses in a gauge invariant way, via the Higgs mechanism. A remnant scalar field, the Higgs boson, is part of the physical spectrum. This is the only missing piece of the Standard Model that still awaits experimental confirmation.

In this course, we intend to give a quite pedestrian introduction to the main concepts involved in the construction of the Standard Model of electroweak interactions. We should not touch any subject "beyond the Standard Model". This primer should provide the necessary background for the lectures on more advanced topics that were covered in this school, such as W physics and extensions of the Standard Model. A special emphasis will be given to the historical aspects of the formulation of the theory. The interplay of new ideas and experimental results make the history of weak interactions a very fruitful laboratory for understanding how the development of a scientific theory works in practice. More formal aspects and details of the model can be found in the vast literature on this subject, from textbooks [1, 2, 3, 4, 5, 6, 7] to reviews [8, 9, 10, 11].

We start these lectures with a chronological account of the ideas related to the development of electromagnetic and weak theories (Section 1.1). The gauge principle (Sec. 1.2) and the concepts of spontaneous symmetry breaking (Sec. 1.3) and the Higgs mechanism (Section 1.4) are presented. In the Chapter 2, we introduce the Standard Model, following the general principles that should guide the construction of a gauge theory. We discuss topics like the mass matrix of the neutral bosons, the measurement of the Weinberg angle, the lepton mass, anomaly cancelation, and the introduction of quarks in the model. We finalize this chapter giving an overview on the Standard Model Lagrangian in Sec. 2.4. In Chapter 3, we give an introduction to the radiative corrections to the Standard Model. Loop calculations are important to compare the predictions of the Standard Model with the precise experimental results of Z physics that are presented in Sec. 3.2. We finish our lectures with an account on the most important challenge to the Standard Model: the discovery of the Higgs boson. In Chapter 4, we discuss the main properties of the Higgs, like mass, couplings and decay modes and discuss the phenomenological prospects for the search of the Higgs in different colliders.

Most of the material covered in these lectures can be found in a series of very good textbook on the subject. Among them we can point out the books from Quigg [1], Aitchison and Hey [4], and Leader and Predazzi [7].

1.1 A Chronology of the Weak Interactions

We will present in this section the main steps given towards a unified description of the electromagnetic and weak interactions. In order to give a historical flavor to the presentation, we will mention some parallel achievements in Particle Physics in this century, from theoretical developments and predictions to experimental confirmation and surprises. The topics closely related to the evolution and construction of the model will be worked with more details.

The chronology of the developments and discoveries in Particle Physics can be found in the books of Cahn and Goldhaber [12] and the annotated bibliography from COMPAS and Particle Data Groups [13]. An extensive selection of original papers on Quantum Electrodynamics can be found in the book edited by Schwinger [14]. Original papers on gauge theory of weak and electromagnetic interactions appear in Ref. [15].

$\boxed{\textbf{\textit{1896}}}$ ⋆ Becquerel [16]: evidence for spontaneous radioactivity effect in uranium decay, using photographic film.

$\boxed{\textbf{\textit{1897}}}$ ⋆ Thomson: discovery of the electron in cathode rays.

$\boxed{\textbf{\textit{1900}}}$ ⋆ Planck: start of the quantum era.

$\boxed{\textbf{\textit{1905}}}$ Einstein: start of the relativistic era.

$\boxed{\textbf{\textit{1911}}}$ ⋆ Millikan: measurement of the electron charge.

$\boxed{\textbf{\textit{1911}}}$ Rutherford: evidence for the atomic nucleus.

$\boxed{\textbf{\textit{1913}}}$ ⋆ Bohr: invention of the quantum theory of atomic spectra.

$\boxed{\textbf{\textit{1914}}}$ Chadwick [17]: first observation that the β spectrum is continuous. Indirect evidence on the existence of neutral penetrating particles.

$\boxed{\textbf{\textit{1919}}}$ Rutherford: discovery of the proton, constituent of the nucleus.

$\boxed{\textbf{\textit{1923}}}$ ⋆ Compton: experimental confirmation that the photon is an elementary particle in $\gamma + C \to \gamma + C$.

⋆The "star" ($\star$) means that the author(s) have received the Nobel Prize in Physics for this particular work.

$\boxed{\mathbf{1923}}$ * de Broglie: corpuscular–wave dualism for electrons.

$\boxed{\mathbf{1925}}$ * Pauli: discovery of the exclusion principle.

$\boxed{\mathbf{1925}}$ * Heisenberg: foundation of quantum mechanics.

$\boxed{\mathbf{1926}}$ * Schrödinger: creation of wave quantum mechanics.

$\boxed{\mathbf{1927}}$ Ellis and Wooster [18]: confirmation that the β spectrum is continuous.

$\boxed{\mathbf{1927}}$ Dirac [19]: foundations of Quantum Electrodynamics (QED).

$\boxed{\mathbf{1928}}$ * Dirac: discovery of the relativistic wave equation for electrons; prediction of the magnetic moment of the electron.

$\boxed{\mathbf{1929}}$ Skobelzyn: observation of cosmic ray showers produced by energetic electrons in a cloud chamber.

$\boxed{\mathbf{1930}}$ Pauli [20]: first proposal, in an open letter, of the existence of a light, neutral and feebly interacting particle emitted in β decay.

$\boxed{\mathbf{1930}}$ Oppenheimer [21]: self–energy of the electron: the first ultraviolet divergence in QED.

$\boxed{\mathbf{1931}}$ Dirac: prediction of the positron and anti–proton.

$\boxed{\mathbf{1932}}$ * Anderson: first evidence for the positron.

$\boxed{\mathbf{1932}}$ * Chadwick: first evidence for the neutron in $\alpha + Be \to C + n$.

$\boxed{\mathbf{1932}}$ Heisenberg: suggestion that nuclei are composed of protons and neutrons.

$\boxed{\mathbf{1934}}$ Pauli [22]: explanation of continuous electron spectrum of β decay — proposal for the neutrino.

$$n \to p + e^{-} + \bar{\nu}_e \ .$$

$\boxed{\mathbf{1934}}$ Fermi [23]: field theory for β decay, assuming the existence of the neutrino. In analogy to "the theory of radiation that describes the emission of a quantum of light from an excited atom", $eJ_\mu A^\mu$, Fermi proposed a current–current Lagrangian to describe the β decay:

$$\mathcal{L}_{\text{weak}} = \frac{G_F}{\sqrt{2}} \left(\bar{\psi}_p \, \gamma_\mu \, \psi_n\right) \left(\bar{\psi}_e \, \gamma^\mu \, \psi_\nu\right) \ .$$

$\boxed{\textbf{\textit{1936}}}$ Gamow and Teller [24]: proposed an extension of the Fermi theory to describe also transitions with $\Delta J^{\rm nuc} \neq 0$. The vector currents proposed by Fermi are generalized to:

$$\mathcal{L}_{\rm weak} = \frac{G_F}{\sqrt{2}} \sum_i C_i \left(\bar{\psi}_p \Gamma^i \psi_n\right) \left(\bar{\psi}_e \Gamma^i \psi_\nu\right) \ ,$$

with the scalar, pseudo–scalar, vector, axial and tensor structures:

$$\Gamma^S = 1\,, \quad \Gamma^P = \gamma_5\,, \quad \Gamma^V_\mu = \gamma_\mu\,, \quad \Gamma^A_\mu = \gamma_\mu\gamma_5\,, \quad \Gamma^T_{\mu\nu} = \sigma_{\mu\nu}\ .$$

Nuclear transitions with $\Delta J = 0$ are described by the interactions $S.S$ and/or $V.V$, while $\Delta J = 0, \pm 1$ $(0 \not\to 0)$ transitions can be taken into account by $A.A$ and/or $T.T$ interactions ($\Gamma^P \to 0$ in the non–relativistic limit). However, interference between them are proportional to m_e/E_e and should increase the emission of low energy electrons. Since this behavior was not observed, the weak Lagrangian should contain,

$$S.S \ \ \underline{\text{or}} \ \ V.V \quad \underline{\text{and}} \quad A.A \ \ \underline{\text{or}} \ \ T.T\ .$$

$\boxed{\textbf{\textit{1937}}}$ Neddermeyer and Anderson: first evidence for the muon.

$\boxed{\textbf{\textit{1937}}}$ Majorana: Majorana neutrino theory.

$\boxed{\textbf{\textit{1937}}}$ Bloch and Nordsieck [25]: treatment of infrared divergences.

$\boxed{\textbf{\textit{1940}}}$ Williams and Roberts [26]: first observation of muon decay

$$\mu^- \to e^- + (\bar{\nu}_e + \nu_\mu)\ .$$

$\boxed{\textbf{\textit{1943}}}$ Heisenberg: invention of the S–matrix formalism.

$\boxed{\textbf{\textit{1943}}}$ * Tomonaga [27]: creation of the covariant quantum electrodynamic theory.

$\boxed{\textbf{\textit{1947}}}$ Pontecorvo [28]: first idea about the universality of the Fermi weak interactions *i.e.* decay and capture processes have the same origin.

$\boxed{\textbf{\textit{1947}}}$ Bethe [29]: first theoretical calculation of the Lamb shift in non–relativistic QED.

$\boxed{\textbf{\textit{1947}}}$ * Kusch and Foley [30]: first measurement of $g_e - 2$ for the electron using the Zeeman effect: $g_e = 2(1 + 1.19 \times 10^{-3})$.

$\boxed{\textbf{\textit{1947}}}$ * Lattes, Occhialini and Powell: confirmation of the π^- and first evidence for pion decay $\pi^\pm \to \mu^\pm + (\nu_\mu)$.

$\boxed{\textbf{\textit{1947}}}$ Rochester and Butler: first evidence for V events (strange particles).

$\boxed{\textbf{\textit{1948}}}$ Schwinger [31]: first theoretical calculation of $g_e - 2$ for the electron: $g_e = 2(1 + \alpha/2\pi) = 2(1 + 1.16 \times 10^{-3})$. The high–precision measurement of the anomalous magnetic moment of the electron is the most stringent QED test. The present theoretical and experimental value of $a_e = (g_e - 2)/2$, are [32],

$$\begin{aligned} a_e^{\rm thr} &= (115\ 965\ 215.4 \pm 2.4) \times 10^{-11} \ , \\ a_e^{\rm exp} &= (115\ 965\ 219.3 \pm 1.0) \times 10^{-11} \ , \end{aligned}$$

where we notice the impressive agreement at the 9 digit level!

$\boxed{\textbf{\textit{1948}}}$ * Feynman [33]; Schwinger [34]; Tati and Tomonaga [35]: creation of the covariant theory of QED.

$\boxed{\textbf{\textit{1949}}}$ Dyson [36]: covariant QED and equivalence of Tomonaga, Schwinger and Feynman methods.

$\boxed{\textbf{\textit{1949}}}$ Wheeler and Tiomno [37]; Lee, Rosenbluth and Yang [38]: proposal of the universality of the Fermi weak interactions. Different processes like,

$$\begin{aligned} \beta - \text{decay} &\quad : \quad n \to p + e^- + \bar\nu_e \ , \\ \mu - \text{decay} &\quad : \quad \mu^- \to e^- + \bar\nu_e + \nu_\mu \ , \\ \mu - \text{capture} &\quad : \quad \mu^- + p \to \nu_\mu + n \ , \end{aligned}$$

must have the same nature and should share the same coupling constant,

$$G_F = \frac{1.03 \times 10^{-5}}{M_p^2} \ ,$$

the so–called Fermi constant.

$\boxed{\textbf{\textit{50's}}}$ A large number of new particles where discovered in the 50's: π^0, $K^\pm$, Λ, K^0, Δ^{++}, Ξ^-, $\Sigma^\pm$, $\bar\nu_e$, $\bar p$, $K_{L,S}$, $\bar n$, Σ^0, $\bar\Lambda$, Ξ^0, $\cdots$

$\boxed{\textbf{\textit{1950}}}$ Ward [39]: Ward identity in QED.

$\boxed{\textbf{\textit{1953}}}$ Stückelberg; Gell–Mann: invention and exploration of renormalization group.

$\boxed{\textbf{\textit{1954}}}$ Yang and Mills [40]: introduction of local gauge isotopic invariance in quantum field theory. This was one of the key theoretical developments that lead to the invention of non–abelian gauge theories.

$\boxed{\textbf{\textit{1955}}}$ Alvarez and Goldhaber [41]; Birge *et al.* [42]: $\theta - \tau$ puzzle: The "two" particles seem to be a single state since they have the same width ($\Gamma_\theta = \Gamma_\tau$), and the same mass ($M_\theta = M_\tau$). However the observation of different decay modes, into states with opposite parity:

$$\begin{aligned} \theta^+ &\to \pi^+ + \pi^0 \,, \qquad\quad J^P = 0^+ \,, \\ \tau^+ &\to \pi^+ + \pi^+ + \pi^- \,, \;\; J^P = 0^- \,, \end{aligned}$$

suggested that parity could be violated in weak transitions.

$\boxed{\textbf{\textit{1955}}}$ Lehmann, Symanzik and Zimmermann: beginnings of the axiomatic field theory of the S–matrix.

$\boxed{\textbf{\textit{1955}}}$ Nishijima: classification of strange particles and prediction of Σ^0 and Ξ^0.

$\boxed{\textbf{\textit{1956}}}$ * Lee and Yang [43]: proposals to test spatial parity conservation in weak interactions.

$\boxed{\textbf{\textit{1957}}}$ Wu *et al.* [44]: obtained the first evidence for parity nonconservation in weak decays. They measured the angular distribution of the electrons in β decay,

$$^{60}\text{Co (polarized)} \to \; ^{60}\text{Ni} + e^- + \bar{\nu}_e \,,$$

and observed that the decay rate depend on the pseudo–scalar quantity: $< \vec{J}_{\text{nuc}} > . \, \vec{p}_e$.

$\boxed{\textbf{\textit{1957}}}$ Garwin, Lederman and Weinrich [45]; Friedman and Telegdi [46]: confirmation of parity violation in weak decays. They make the measurement of the electron asymmetry (muon polarization) in the decay chain,

$$\begin{aligned} \pi^+ &\to \mu^+ + \nu_\mu \\ &\qquad \hookrightarrow e^+ + \nu_e + \bar{\nu}_\mu \,. \end{aligned}$$

1957 Frauenfelder *et al.* [47]: further confirmation of parity nonconservation in weak decays. The measurement of the longitudinal polarization of the electron ($\vec{\sigma}_e.\vec{p}_e$) emitted in β decay,

$$^{60}\text{Co} \rightarrow \ e^- \ (\text{long. polar.}) + \bar{\nu}_e + X \ ,$$

showed that the electrons emitted in weak transitions are mostly left–handed.

The confirmation of the parity violation by the weak interaction showed that it is necessary to have a term containing a γ_5 in the weak current:

$$\mathcal{L}_{\text{weak}} \ \rightarrow \ \frac{G_F}{\sqrt{2}} \sum_i C_i \left(\bar{\psi}_p \, \Gamma^i \, \psi_n \right) \left[\bar{\psi}_e \, \Gamma^i \, (1 \pm \gamma_5) \, \psi_\nu \right] \ .$$

Note that CP remains conserved since C is also violated.

1957 Salam [48] ; Lee and Yang [49]; Landau [50]: two–component theory of neutrino. This requires that the neutrino is either right <u>or</u> left–handed.

Since it was known that electrons (positrons) involved in weak decays are left (right) handed, the leptonic current should be written as:

$$J^i_{\text{lept}} \equiv \left[\bar{\psi}_e \, \Gamma^i \, (1 \pm \gamma_5) \, \psi_\nu \right] \rightarrow \left[\bar{\psi}_e \, \frac{(1 + \gamma_5)}{2} \, \Gamma^i \, (1 \pm \gamma_5) \, \psi_\nu \right] \ .$$

Therefore the measurement of the neutrino helicity is crucial to determine the structure of the weak current. If Γ^i = V or A then $\{\gamma_5, \Gamma^i\} = 0$ and the neutrino should be left–handed, otherwise the current is zero. On the other hand, if Γ^i = S or T, then $[\gamma_5, \Gamma^i] = 0$, and the neutrino should be right–handed.

1957 Schwinger [51]; Lee and Yang [52]: development of the idea of the intermediate vector boson in weak interaction. The four–fermion Fermi interaction is "point–like" *i.e.* a s–wave interaction. Partial wave unitarity requires that such interaction must give rise to a cross section that is bound by $\sigma < 4\pi/p^2_{\text{cm}}$. However, since G_F has dimension of M^{-2}, the cross section for the Fermi weak interaction should go like $\sigma \sim G_F^2 p^2_{\text{cm}}$. Therefore the Fermi theory violates unitarity for $p_{\text{cm}} \simeq 300$ GeV.

This violation can be delayed by imposing that the interaction is transmitted by a intermediate vector boson (IVB) in analogy, once again, with the quantum electrodynamics. Here, the IVB should have quite different characteristics, due to the properties of the weak interaction. The IVB should be charged since the β decay requires charge–changing currents. They should also be very massive to account for short range of the weak interaction and they should not have a definite parity to allow, for instance, a $V - A$ structure for the weak current.

With the introduction of the IVB, the Fermi Lagrangian for leptons,

$$\mathcal{L}_{\text{weak}} = \frac{G_F}{\sqrt{2}} \left[J^\alpha(\ell) J^\dagger_\alpha(\ell') + \text{h.c.} \right] \ ,$$

where $J^\alpha(\ell) = \bar{\psi}_{\nu_\ell} \Gamma^\alpha \psi_\ell$, becomes:

$$\mathcal{L}^W_{\text{weak}} = G_W \left(J^\alpha W^+_\alpha + J^{\dagger\,\alpha} W^-_\alpha \right) \ , \tag{1.1}$$

with a new coupling constant G_W.

Let us compare the invariant amplitude for μ–decay, in the low–energy limit in both cases. For the Fermi Lagrangian, we have,

$$\mathcal{M}_{\text{weak}} = i \frac{G_F}{\sqrt{2}} J^\alpha(\mu) J_\alpha(e) \ . \tag{1.2}$$

On the other hand, when we take into account the exchange of the IVB, the invariant amplitude should include the vector boson propagator,

$$\mathcal{M}^W_{\text{weak}} = \left[i\, G_W J^\alpha(\mu) \right] \left[\frac{-i}{k^2 - M_W^2} \left(g_{\alpha\beta} - \frac{k_\alpha k_\beta}{M_W^2} \right) \right] \left[i\, G_W J^\beta(e) \right] \ .$$

At low energies, *i.e.* for $k^2 \ll M_W^2$,

$$\mathcal{M}^W_{\text{weak}} \longrightarrow i \frac{G_W^2}{M_W^2} J^\alpha(\mu) J_\alpha(e) \ , \tag{1.3}$$

and, comparing (1.3) with (1.2) we obtain the relation

$$\boxed{G_W^2 = \frac{M_W^2 G_F}{\sqrt{2}}} \ , \tag{1.4}$$

which shows that G_W is dimensionless.

However, at high energies, the theory of IVB still violates unitarity, for instance, in the cross section for $\nu\bar{\nu} \to W^+W^-$ (see Fig. 1).

Let us consider the $W^\pm$ polarization states. At the $W^\pm$ rest frame, we can define the transversal and longitudinal polarizations as

$$\begin{aligned} \epsilon^\mu_{T_1}(0) &= (0,1,0,0)\ , \\ \epsilon^\mu_{T_2}(0) &= (0,0,1,0)\ , \\ \epsilon^\mu_L(0) &= (0,0,0,1)\ . \end{aligned}$$

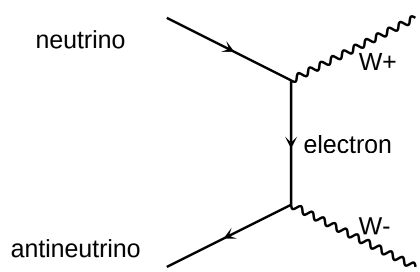


Fig. 1: Feynman diagram for the process $\nu + \bar{\nu} \to W^+ + W^-$.

After a boost along the z direction, *i.e.* for $p^\mu = (E,0,0,p)$, the transversal states remain unchanged while the longitudinal state becomes,

$$\epsilon^\mu_L(p) = \left(\frac{|\vec{p}|}{M_W}, \frac{E}{M_W}\hat{p}\right) \simeq \frac{p^\mu}{M_W}\ .$$

Since the longitudinal polarization is proportional to the vector boson momentum, at high energies the longitudinal amplitudes should give rise to the worst behavior.

In fact, in high energy limit, the polarized cross section for $\nu\bar{\nu} \to W^+W^-$ behaves like,

$$\begin{aligned} \sigma(\nu\bar{\nu} \to W^+_T W^-_T) &\longrightarrow \text{constant} \\ \sigma(\nu\bar{\nu} \to W^+_L W^-_L) &\longrightarrow \frac{G_F^2 s}{3\pi}\ , \end{aligned}$$

which still violates unitarity for large values of s.

1958 Feynman and Gell–Mann [53]; Marshak and Sudarshan [54]; Sakurai [55]: universal $V-A$ weak interactions.

$$J^{+\,\mu}_{\rm lept} = \left[\bar{\psi}_e\,\gamma^\mu(1-\gamma_5)\,\psi_\nu\right]\ . \tag{1.5}$$

1958 Leite Lopes [56]: hypothesis of neutral vector mesons exchanged in weak interaction. Prediction of its mass of $\sim 60\ m_{\rm proton}$.

1958 Goldhaber, Grodzins and Sunyar [57]: first evidence for the negative ν_e helicity. As mentioned before, this result requires that the structure of the weak interaction is $V-A$.

1959 * Reines and Cowan: confirmation of the detection of the $\bar{\nu}_e$ in $\bar{\nu}_e + p \to e^+ + n$.

1961 Goldstone [58]: prediction of unavoidable massless bosons if global symmetry of the Lagrangian is spontaneously broken.

1961 Salam and Ward [59]: invention of the gauge principle as basis to construct quantum field theories of interacting fundamental fields.

1961 * Glashow [60]: first introduction of the neutral intermediate weak boson (Z^0).

1962 * Danby *et al.*: first evidence of ν_μ from $\pi^\pm \to \mu^\pm + (\nu/\bar{\nu})$.

1963 Cabibbo [61]: introduction of the Cabibbo angle and hadronic weak currents.

It was observed experimentally that weak decays with change of strangeness ($\Delta s = 1$) are strongly suppressed in nature. For instance, the width of the neutron is much larger than the Λ's,

$$\Gamma_{\Delta s=0}\left(n_{udd} \to p_{uud}\ e\bar{\nu}\right) \gg \Gamma_{\Delta s=1}\left(\Lambda_{uds} \to p_{uud}\ e\bar{\nu}\right)\ ,$$

which yield a branching ratio of 100% in the case of neutron and just $\sim 8\times 10^{-4}$ for the Λ.

The hadronic current, in analogy with leptonic current (1.5), can be written in terms of the u, d, and s quarks,

$$J^H_\mu = \bar{d}\gamma_\mu(1-\gamma_5)u + \bar{s}\gamma_\mu(1-\gamma_5)u\ , \tag{1.6}$$

where the first term is responsible for the $\Delta s = 0$ transitions while the latter one gives rise to the $\Delta s = 1$ processes. In order to make the hadronic current also universal, with a common coupling constant G_F, Cabibbo introduced a mixing angle to give the right weight to the $\Delta s = 0$ and $\Delta s = 1$ parts of the hadronic current,

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} , \qquad (1.7)$$

where d', s' (d, s) are interaction (mass) eigenstates. Now the transition $\bar{d} \leftrightarrow u$ is proportional to $G_F \cos\theta_C \simeq 0.97\ G_F$ and the $\bar{s} \leftrightarrow u$ goes like $G_F \sin\theta_C \simeq 0.24\ G_F$.

The hadronic current should now be given in terms of the new interaction eigenstates,

$$\begin{aligned} J_\mu^H &= \bar{d}'\gamma_\mu(1-\gamma_5)u \\ &= \cos\theta_C\ \bar{d}\gamma_\mu(1-\gamma_5)u + \sin\theta_C\ \bar{s}\gamma_\mu(1-\gamma_5)u\ . \end{aligned} \qquad (1.8)$$

_1964_ Bjorken and Glashow [62]: proposal for the existence of a charmed fundamental fermion (c).

_1964_ Higgs [63]; Englert and Brout [64]; Guralnik, Hagen and Kibble [65]: example of a field theory with spontaneous symmetry breakdown, no massless Goldstone boson, and massive vector boson.

_1964_ $\star$ Christenson, Cronin, Fitch and Turlay [66]: first evidence of CP violation in the decay of K^0 mesons.

_1964_ $\star$ Salam and Ward [67]: Lagrangian for the electroweak synthesis, estimation of the W mass.

_1964_ $\star$ Gell–Mann; Zweig: introduction of quarks as fundamental building blocks for hadrons.

_1964_ Greenberg; Han and Nambu: introduction of color quantum number and colored quarks and gluons.

_1967_ Kibble [68]: extension of the Higgs mechanism of mass generation for non–abelian gauge field theories.

_1967_ $\star$ Weinberg [69]: Lagrangian for the electroweak synthesis and estimation of W and Z masses.

$\boxed{\textbf{\textit{1967}}}$ Faddeev and Popov [70]: method for construction of Feynman rules for Yang–Mills gauge theories.

$\boxed{\textbf{\textit{1968}}}$ * Salam [71]: Lagrangian for the electroweak synthesis.

$\boxed{\textbf{\textit{1969}}}$ Bjorken: invention of the Bjorken scaling behavior.

$\boxed{\textbf{\textit{1969}}}$ Feynman: birth of the partonic picture of hadron collisions.

$\boxed{\textbf{\textit{1970}}}$ Glashow, Iliopoulos and Maiani [72]: introduction of lepton–quark symmetry and the proposal of charmed quark (GIM mechanism).

$\boxed{\textbf{\textit{1971}}}$ * 't Hooft [73]: rigorous proof of renormalizability of the massless and massive Yang– Mills quantum field theory with spontaneously broken gauge invariance.

$\boxed{\textbf{\textit{1973}}}$ Kobayashi and Maskawa [74]: CP violation is accommodated in the Standard Model with six favours.

$\boxed{\textbf{\textit{1973}}}$ Hasert *et al.* (CERN) [75]: first experimental indication of the existence of weak neutral currents.

$$\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^- \quad , \quad \nu_\mu + N \to \nu_\mu + X \ .$$

This was a dramatic prediction of the Standard Model and its discovery was a major success for the model. They also measured the ratio of neutral–current to charged–current events giving a estimate for the Weinberg angle $\sin^2\theta_W$ in the range 0.3 to 0.4.

$\boxed{\textbf{\textit{1973}}}$ Gross and Wilczek; Politzer: discovery of asymptotic freedom property of interacting Yang–Mills field theories.

$\boxed{\textbf{\textit{1973}}}$ Fritzsch, Gell–Mann and Leutwyler: invention of the QCD Lagrangian.

$\boxed{\textbf{\textit{1974}}}$ Benvenuti *et al.* (Fermilab) [76]: confirmation of the existence of weak neutral currents in the reaction

$$\nu_\mu + N \to \nu_\mu + X \ .$$

$\boxed{\textbf{\textit{1974}}}$ * Aubert *et al.* (Brookhaven); Augustin *et al.* (SLAC): evidence for the J/ψ ($c\bar{c}$).

$\boxed{\textbf{\textit{1975}}}$ * Perl *et al.* (SLAC) [77]: first indication of the τ lepton.

$\boxed{\textbf{\textit{1977}}}$ Herb *et al.* (Fermilab) [78]: first evidence of Υ ($b\bar{b}$).

$\boxed{\textbf{\textit{1979}}}$ Barber *et al.* (MARK J Collab.); Brandelik *et al.* (TASSO Collab.); Berger *et al.* (PLUTO Collab.); W. Bartel (JADE Collab.): evidence for the gluon jet in $e^+e^- \to 3$ jet.

$\boxed{\textbf{\textit{1983}}}$ * Arnison *et al.* (UA1 Collab.) [79]; Banner *et al.* (UA2 Collab.) [80]: evidence for the charged intermediate bosons $W^\pm$ in the reactions

$$p + \bar{p} \to W(\to \ell + \nu) + X \ .$$

They were able to estimate the W boson mass ($M_W = 81 \pm 5$ GeV) in good agreement with the predictions of the Standard Model.

$\boxed{\textbf{\textit{1983}}}$ * Arnison *et al.* (UA1 Collab.) [81]; Bagnaia *et al.* (UA2 Collab.) [82]: evidence for the neutral intermediate boson Z^0 in the reaction

$$p + \bar{p} \to Z(\to \ell^+ + \ell^-) + X \ .$$

This was another important confirmation of the electroweak theory.

$\boxed{\textbf{\textit{1986}}}$ * Van Dyck, Schwinberg and Dehmelt [83]: high precision measurement of the electron $g_e - 2$ factor.

$\boxed{\textbf{\textit{1987}}}$ Albrecht *et al.* (ARGUS Collab.) [84]: first evidence of $B^0 - \bar{B}^0$ mixing.

$\boxed{\textbf{\textit{1989}}}$ Abrams *et al.* (MARK-II Collab.) [85]: first evidence that the number of light neutrinos is 3.

$\boxed{\textbf{\textit{1992}}}$ LEP Collaborations (ALEPH, DELPHI, L3 and OPAL) [86]: precise determination of the Z^0 parameters.

$\boxed{\textbf{\textit{1995}}}$ Abe *et al.* (CDF Collab.) [87]; Abachi *et al.* (DØ Collab.) [88]: observation of the top quark production.

1.2 The Gauge Principle

As it is well known, symmetry has always played a very important rôle in the development of physics. From the spacetime symmetry of special relativity, up to the internal and gauge invariances, the symmetries have mapped out the route to most of the physical theories in this last century.

An important result for field theory and particle physics is provided by the Noether's theorem. If an action is invariant under some group of transformations (symmetry), then there exist one or more conserved quantities (constants of motion) which are associated to these transformations. In this sense, Noether's theorem establishes that symmetries imply conservation laws.

A natural question to ask would be: upon imposing to a given Lagrangian the invariance under a certain symmetry, would it be possible to determine the form of the interaction among the particles? In other words, could symmetry also imply dynamics?

In fact, this happens in Quantum Electrodynamics (QED), the best theory ever built to describe Nature, which had become a prototype of a successful quantum field theory. In QED the existence and some of the properties of the gauge field — the photon — follow from a principle of invariance under *local gauge transformations* of the $U(1)$ group.

Could this principle be generalized to other interactions? For Salam and Ward [59], who invented the gauge principle as the basis to construct the quantum field theory of interacting fields, this was a possible dream:

> *"Our basic postulate is that it should be possible to generate strong, weak and electromagnetic interaction terms (with all their correct symmetry properties and also with clues regarding their relative strengths) by making local gauge transformations on the kinetic–energy terms in the free Lagrangian for all particles."*

In fact, those ideas could be accomplished just after some new and important ingredients were introduced to describe short distance (weak)

and strong interactions. In the case of weak interactions the presence of very heavy weak gauge bosons require the new concept of spontaneous breakdown of the gauge symmetry and the Higgs mechanism [63, 64, 65]. On the other hand, the concept of asymptotic freedom [89, 90] played a crucial rôle to describe perturbatively the strong interaction at short distances, making the strong gauge bosons trapped. The Quantum Chromodynamics (QCD), the gauge theory for strong interactions, is the subject of Mangano's lecture at this school.

1.2.1 Gauge Invariance in Quantum Mechanics

The gauge principle and the concept of gauge invariance are already present in Quantum Mechanics of a particle in the presence of an electromagnetic field [4]. Let us start from the classical Hamiltonian that gives rise to the Lorentz force ($\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$),

$$\mathcal{H} = \frac{1}{2m}\left(\vec{p} - q\vec{A}\right)^2 + q\phi \ , \tag{1.9}$$

where the electric and magnetic fields can be described in terms of the potentials $A^\mu = (\phi, \vec{A})$,

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}; \ , \quad \vec{B} = \vec{\nabla} \times \vec{A} \ .$$

These fields remain exactly the same when we make the *gauge transformation* (G) in the potentials:

$$\phi \to \phi' = \phi - \frac{\partial \chi}{\partial t} \quad , \quad \vec{A} \to \vec{A}' = \vec{A} + \vec{\nabla}\chi \ . \tag{1.10}$$

When we quantize the Hamiltonian (1.9) by applying the usual prescription $\vec{p} \to -i\vec{\nabla}$, we get the Schrödinger equation for a particle in an electromagnetic field,

$$\left[\frac{1}{2m}\left(-i\vec{\nabla} - q\vec{A}\right)^2 + q\phi\right]\psi(x,t) = i\frac{\partial \psi(x,t)}{\partial t} \ ,$$

which can be written in a compact form as

$$\frac{1}{2m}(-i\vec{D})^2\psi = iD_0\psi \ , \tag{1.11}$$

The equation (1.11) is equivalent to make the substitution

$$\vec{\nabla} \to \vec{D} = \vec{\nabla} - iq\vec{A} \ , \quad \frac{\partial}{\partial t} \to D_0 = \frac{\partial}{\partial t} + iq\phi \ .$$

in the free Schrödinger equation.

If we make the gauge transformation, $(\phi, \vec{A}) \xrightarrow{G} (\phi', \vec{A}')$, given by (1.10), does the new field ψ' which is solution of

$$\frac{1}{2m}(-i\vec{D}')^2 \ \psi' = iD_0' \, \psi' \ ,$$

describe the same physics?

The answer to this question is *no*. However, we can recover the invariance of our theory by making, at the same time, the phase transformation in the matter field

$$\psi' = \exp{(iq\chi)}\, \psi \tag{1.12}$$

with the same function $\chi = \chi(x,t)$ used in the transformation of electromagnetic fields (1.10). The derivative of ψ' transforms as,

$$\begin{aligned} \vec{D}'\psi' &= \left[\vec{\nabla} - iq(\vec{A} + \vec{\nabla}\chi)\right] \exp{(iq\chi)}\, \psi \\ &= \exp{(iq\chi)}\, (\vec{\nabla}\psi) + iq(\vec{\nabla}\chi) \exp{(iq\chi)}\, \psi \\ &\quad -iq\vec{A} \exp{(iq\chi)}\, \psi - iq(\vec{\nabla}\chi) \exp{(iq\chi)}\, \psi \\ &= \exp{(iq\chi)}\, \vec{D}\psi \ , \end{aligned} \tag{1.13}$$

and in the same way, we have for D_0,

$$D_0'\psi' = \exp{(iq\chi)}\, D_0\psi \ . \tag{1.14}$$

We should mention that now the field ψ (1.12) and its derivatives $\vec{D}\psi$ (1.13), and $D_0\psi$ (1.14), all transform exactly in the same way: they are all multiplied by the same phase factor.

Therefore, the Schrödinger equation (1.11) for ψ' becomes

$$\begin{aligned} \frac{1}{2m}(-i\vec{D}')^2\psi' &= \frac{1}{2m}(-i\vec{D}')(-i\vec{D}'\psi') \\ &= \frac{1}{2m}(-i\vec{D}') \left[-i \exp{(iq\chi)}\, \vec{D}\psi\right] \\ &= \exp{(iq\chi)}\, \frac{1}{2m}(-i\vec{D})^2\psi \\ &= \exp{(iq\chi)}\, (iD_0)\psi = iD_0'\psi' \ . \end{aligned}$$

and now both ψ and ψ' describe the same physics, since $|\psi|^2 = |\psi'|^2$. In order to get the invariance for all observables, we should assure that the following substitution is made:

$$\vec{\nabla} \to \vec{D} \quad , \qquad \frac{\partial}{\partial t} \to D_0 \ ,$$

For instance, the current

$$\vec{J} \propto \psi^* (\vec{\nabla}\psi) - (\vec{\nabla}\psi)^* \psi \ ,$$

becomes also gauge invariant with this substitution since

$$\psi^{*\prime}(\vec{D}'\psi') = \psi^* \exp(-iq\chi) \exp(iq\chi) (\vec{D}\psi) = \psi^* (\vec{D}\psi) \ .$$

After we have shown how to obtain a gauge invariant quantum description of a particle in an electromagnetic field, could we reverse the argument? That is: when we demand that a theory is invariant under a spacetime dependent phase transformation, can this procedure impose the specific form of the interaction with the gauge field? In other words, can the *symmetry imply dynamics*?

Let us examine what happens when we start from the Dirac free Lagrangian

$$\mathcal{L}_\psi = \bar{\psi}(i \not\partial - m)\psi \ ,$$

that is not invariant under the *local gauge transformation*,

$$\psi \to \psi' = \exp[-i\alpha(x)]\, \psi \ ,$$

since

$$\mathcal{L}_\psi \to \mathcal{L}'_\psi = \mathcal{L}_\psi + \bar{\psi}\gamma_\mu \psi (\partial^\mu \alpha) \ ,$$

However, if we introduce the *gauge field* A_μ through the *minimal coupling*

$$D_\mu \equiv \partial_\mu + ieA_\mu \ ,$$

and, at the same time, require that A_μ transforms like

$$A_\mu \to A'_\mu = A_\mu + \frac{1}{e}\partial_\mu \alpha \ . \tag{1.15}$$

we have

$$\begin{aligned}
\mathcal{L}_\psi \to \mathcal{L}'_\psi &= \bar{\psi}' \left[(i\ \partial\!\!\!/ - e\ A\!\!\!/') - m\right]\psi' \\
&= \bar{\psi}\exp(+i\alpha)\left[i\ \partial\!\!\!/ - e\left(A\!\!\!/ + \frac{1}{e}\ \partial\!\!\!/\alpha\right) - m\right]\exp(-i\alpha)\psi \\
&= \mathcal{L}_\psi - e\bar{\psi}\gamma_\mu\psi A^\mu \ . \qquad (1.16)
\end{aligned}$$

The coupling between ψ (*e.g.* electrons) and the gauge field A_μ (photon) arises naturally when we require the invariance under local gauge transformations of the kinetic–energy terms in the free fermion Lagrangian.

Since, the electromagnetic strength tensor

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \ , \qquad (1.17)$$

is invariant under the gauge transformation (1.15), so is the Lagrangian for free gauge field,

$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \ , \qquad (1.18)$$

This Lagrangian together with (1.16) describes the Quantum Electrodynamics.

We should point out that a hypothetical mass term for the gauge field,

$$\mathcal{L}_A^m = -\frac{1}{2}A_\mu A^\mu \ ,$$

is not invariant under the transformation (1.15). Therefore, something else should be necessary to describe massive vector bosons in a gauge invariant way, preserving the renormalizability of the theory.

1.2.2 Gauge Invariance for Non–Abelian Groups

As suggested by Heisenberg [91] in 1932, under nuclear interactions, protons and neutron can be regarded as degenerated since their mass are quite similar and electromagnetic interaction is negligible.

Therefore any arbitrary combination of their wave function would be equivalent,

$$\psi \equiv \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} \to \psi' = U\psi \ ,$$

where U is unitary transformation ($U^\dagger U = UU^\dagger = 1$) to preserve normalization (probability). Moreover, if det$|U|$ = 1, U represents the Lie group $SU(2)$:

$$U = \exp\left(-i\frac{\tau^a}{2}\alpha^a\right) \simeq 1 - i\frac{\tau^a}{2}\alpha^a \ ,$$

where τ^a, $a = 1, 2, 3$ are the Pauli matrices.

In 1954, Yang and Mills [40] introduced the idea of local gauge isotopic invariance in quantum field theory.

> *"The differentiation between a neutron and a proton is then a purely arbitrary process. As usually conceived, however, this arbitrariness is subject to the following limitation: once one chooses what to call a proton, what a neutron, at one spacetime point, one is then not free to make any choices at other spacetime points. It seems that this is not consistent with the localized field concept that underlies the usual physical theories."*

Following their argument, we should preserve our freedom to choose what to call a proton or a neutron *no matter when or where we are*. This can be implemented by requiring that the gauge parameters depend on the spacetime points, *i.e.* $\alpha^a \to \alpha^a(x)$.

This idea was generalized by Utiyama [92] in 1956 for any non–Abelian group G with generators t_a satisfying the Lie algebra [8],

$$[t_a, t_b] = i\, C_{abc}\, t_c \ ,$$

with C_{abc} being the structure constant of the group.

The Lagrangian $\mathcal{L}_\psi$ should be invariant under the *matter field transformation*

$$\psi \to \psi' = \Omega\, \psi \ ,$$

with

$$\Omega \equiv \exp\left[-i\,T^a \alpha^a(x)\right] \ ,$$

where T^a is a convenient representation (*i.e.* according to the fields ψ) of the generators t^a.

Introducing one gauge field for each generator, and defining the *covariant derivative* by

$$D_\mu \equiv \partial_\mu - igT^a A^a_\mu \ ,$$

Since the covariant derivative transforms just like the matter field, *i.e.* $D_\mu\psi \rightarrow \Omega\ (D_\mu\psi)$, this will ensure the invariance under the local non–Abelian gauge transformation for the terms containing the fields and its gradients as long as the *gauge field transformation* is

$$T^a A^a_\mu \rightarrow \Omega \left(T^a A^a_\mu + \frac{i}{g}\partial_\mu \right) \Omega^{-1} \ ,$$

or, in infinitesimal form, *i.e.* for $\Omega \simeq 1 - i\,T^a\alpha^a(x)$,

$$A^{a\,\prime}_\mu = A^a_\mu - \frac{1}{g}\partial_\mu\alpha^a + C_{abc}\,\alpha^b A^c_\mu \ .$$

Finally, we should generalize the *strength tensor* (1.17) for a non–abelian Lie group,

$$F^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,C_{abc}\,A^b_\mu A^c_\nu \ , \tag{1.19}$$

which transforms like $F^{a\,\prime}_{\mu\nu} \rightarrow F^a_{\mu\nu} + C_{abc}\alpha^b F^c_{\mu\nu}$. Therefore, the invariant kinetic term for the gauge bosons, can be written as

$$\mathcal{L}_A = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} \ , \tag{1.20}$$

and is invariant under the local gauge transformation. However, a *mass term* for the gauge bosons like

$$A^a_\mu A^{a\,\mu} \rightarrow \left(A^a_\mu - \frac{1}{g}\partial_\mu\alpha^a + C_{abc}\alpha^b A^c_\mu \right) \left(A^{a\,\mu} - \frac{1}{g}\partial_\mu\alpha^a + C_{ade}\alpha^d A^{e\,\mu} \right) \ ,$$

is still *not* gauge invariant.

Note that since

$$F \propto (\partial A - \partial A) + gAA \ ,$$

unlike the Abelian case, there is a new feature: the gauge fields have triple and quartic *self–couplings*,

$$\mathcal{L}_A \propto \underbrace{(\partial A - \partial A)^2}_{\text{propagator}} + \underbrace{g(\partial A - \partial A)AA}_{\text{triple}} + \underbrace{g^2 AAAA}_{\text{quartic}} \ .$$

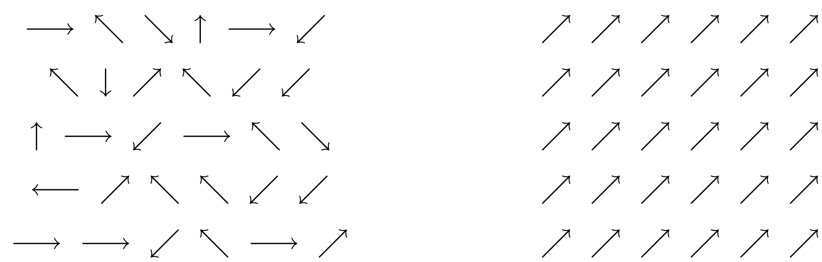
1.3 Spontaneous Symmetry Breaking

Exact symmetries give rise, in general, to exact conservation laws. In this case both the Lagrangian and the vacuum (the ground state of the theory) are invariant. However, there are some conservation laws which are not exact, *e.g.* isospin, strangeness, etc. These situations can be described by adding to the invariant Lagrangian ($\mathcal{L}_{\rm sym}$) a small term that breaks this symmetry ($\mathcal{L}_{\rm sb}$),

$$\mathcal{L} = \mathcal{L}_{\rm sym} + \varepsilon \, \mathcal{L}_{\rm sb} \ .$$

Another situation occurs when the system has a Lagrangian that is invariant and a non–invariant vacuum. A classic example of the situation is provided by a ferromagnet where the Lagrangian describing the spin–spin interaction is invariant under tridimensional rotations. For temperatures above the ferromagnetic transition temperature (T_C) the spin system is completely disordered (paramagnetic phase), and therefore the vacuum is also $SO(3)$ invariant [see Fig. 2(a)].

However, for temperatures below T_C (ferromagnetic phase) a spontaneous magnetisation of the system occurs, aligning the spins in some specific direction [see Fig. 2(b)]. In this case, the vacuum is not invariant under the $SO(3)$ group. This symmetry is broken to $SO(2)$, representing the rotation of the whole system around the spin directions.



(a) (b)

Fig. 2: Representation of the spin orientation in the paramagnetic (a) and ferromagnetic (b) phases.

Let us analyze the simple example of a scalar self–interacting real field with Lagrangian,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\ \partial^\mu\phi - V(\phi)\ , \tag{1.21}$$

with

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\ . \tag{1.22}$$

In the theory of the phase transition of a ferromagnet, the Gibbs free energy density is analogous to $V(\phi)$ with ϕ playing the rôle of the average spontaneous magnetisation M.

The whole Lagrangian (1.21) is invariant under the discrete transformation

$$\phi \to -\phi\ . \tag{1.23}$$

Is the vacuum also invariant under this transformation? The vacuum (ϕ_0) can be obtained from the Hamiltonian

$$\mathcal{H} = \frac{1}{2}\left[(\partial_0\phi)^2 + (\nabla\phi)^2\right] + V(\phi)\ .$$

We notice that $\phi_0 = constant$ corresponds to the minimum of $V(\phi)$ and consequently of the energy:

$$\phi_0(\mu^2 + \lambda\phi_0^2) = 0\ .$$

Since λ should be positive to guarantee that $\mathcal{H}$ is bounded, the minimum depends on the *sign of* μ. For $\mu^2 > 0$, we have just one vacuum at $\phi_0 = 0$ and it is also invariant under (1.23) [see Fig. 3 (a)]. However, for $\mu^2 < 0$, we have two vacua states corresponding to $\phi_0^{\pm} = \pm\sqrt{-\mu^2/\lambda}$ [see Fig. 3 (b)]. This case corresponds to a wrong sign for the ϕ mass term.

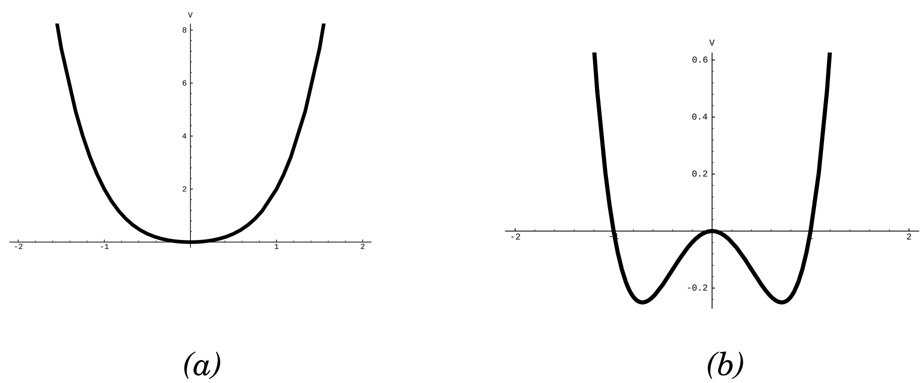


Fig. 3: Scalar potential (1.22) for $\mu^2 > 0$ (a) and for $\mu^2 < 0$ (b).

Since the Lagrangian is invariant under (1.23) the choice between ϕ_0^+ or ϕ_0^- is irrelevant *. Nevertheless, once one choice is made (*e.g.* $v = \phi_0^+$) the symmetry is *spontaneously broken* since $\mathcal{L}$ is invariant but the vacuum is *not*.

Defining a new field ϕ' by shifting the old field by $v = \sqrt{-\mu^2/\lambda}$,

$$\phi' \equiv \phi - v \ ,$$

we verify that the vacuum of the new field is $\phi'_0 = 0$, making the theory suitable for small oscillations around the vacuum state. The Lagrangian becomes:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi' \partial^\mu \phi' - \frac{1}{2}\left(\sqrt{-2\mu^2}\right)^2 \phi'^{\,2} - \lambda\, v\, \phi'^{\,3} - \frac{1}{4}\lambda \phi'^{\,4} \ .$$

This Lagrangian describes a scalar field ϕ' with real and positive mass, $M_{\phi'} = \sqrt{-2\mu^2}$, but it lost the original symmetry due to the $\phi'^{\,3}$ term.

*For an interesting discussion discarding the invariant state $(\phi_0^+ \pm \phi_0^-)$ as the true vacuum see Ref. [93]

A new interesting phenomenon happens when a *continuous* symmetry is spontaneously broken. Let us analyze the case of a charged self–interacting scalar field,

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi) \ , \tag{1.24}$$

with a similar potential,

$$V(\phi^* \phi) = \mu^2 (\phi^* \phi) + \lambda (\phi^* \phi)^2 \ . \tag{1.25}$$

Notice that the Lagrangian (1.24) is invariant under the *global phase* transformation

$$\phi \rightarrow \exp(-i\theta)\phi \ .$$

When we redefine the complex field in terms of two real fields by

$$\phi = \frac{(\phi_1 + i\phi_2)}{\sqrt{2}} \ ,$$

the Lagrangian (1.24) becomes

$$\mathcal{L} = \frac{1}{2} \left(\partial_\mu \phi_1 \partial^\mu \phi_1 + \partial_\mu \phi_2 \partial^\mu \phi_2 \right) - V(\phi_1, \phi_2) \ , \tag{1.26}$$

which is invariant under $SO(2)$ rotations,

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \longrightarrow \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \ .$$

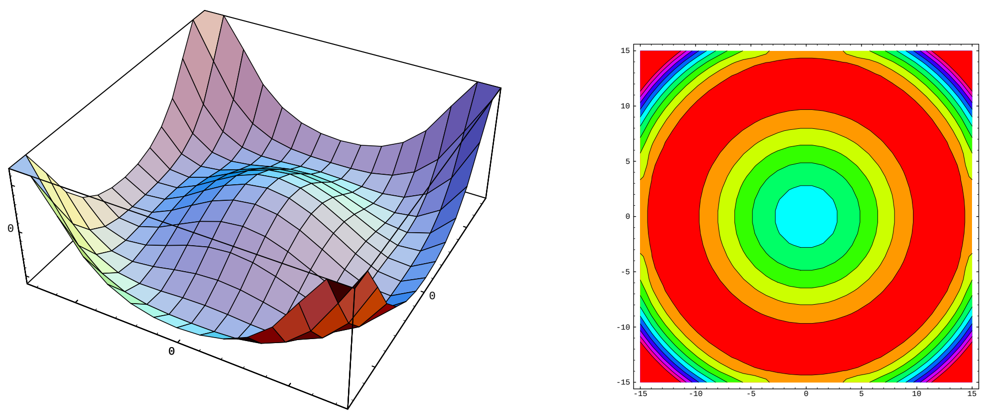
For $\mu^2 > 0$ the vacuum is at $\phi_1 = \phi_2 = 0$, and for small oscillations,

$$\mathcal{L} = \sum_{i=1}^{2} \frac{1}{2} \left(\partial_\mu \phi_i \partial^\mu \phi_i - \mu^2 \phi_i^2 \right) \ ,$$

which means that we have two scalar fields ϕ_1 and ϕ_2 with mass $m^2 = \mu^2 > 0$.

In the case of $\mu^2 < 0$ we have a continuum of distinct vacua [see Fig. 4 (a)] located at

$$< |\phi|^2 >= \frac{(< \phi_1 >^2 + < \phi_2 >^2)}{2} = \frac{-\mu^2}{2\lambda} \equiv \frac{v^2}{2} \ . \tag{1.27}$$



(a) *(b)*

Fig. 4: The potential $V(\phi_1, \phi_2)$ (a) and its contour plot (b)

We can see from the contour plot [Fig. 4 (b)] that the vacua are also invariant under $SO(2)$. However, this symmetry is spontaneously broken when we choose a particular vacuum. Let us choose, for instance, the configuration,

$$\begin{aligned} \phi_1 &= v \ , \\ \phi_2 &= 0 \ . \end{aligned}$$

The new fields, suitable for small perturbations, can be defined as,

$$\begin{aligned} \phi_1' &= \phi_1 - v \ , \\ \phi_2' &= \phi_2 \ . \end{aligned}$$

In terms of these new fields the Lagrangian (1.26) becomes,

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi_1' \partial^\mu \phi_1' - \frac{1}{2}(-2\mu^2){\phi_1'}^2 + \frac{1}{2}\partial_\mu \phi_2' \partial^\mu \phi_2' + \text{ interaction terms} \ .$$

Now we identify in the particle spectrum a scalar field ϕ_1' with real and positive mass and a massless scalar boson (ϕ_2'). This could be seen from Fig. 4 (b), when we consider the mass matrix in tree approximation,

$$M_{ij}^2 = \left. \frac{\partial^2 V(\phi_1', \phi_2')}{\partial \phi_i' \partial \phi_j'} \right|_{\phi' = \phi_0'} .$$

The second derivative of $V(\phi_1', \phi_2')$ in the ϕ_2' direction corresponds to the zero eigenvalue of the mass matrix, while for ϕ_1' it is positive.

This is an example of the prediction of the so called Goldstone theorem [58] which states that when an exact continuous global symmetry is spontaneously broken, *i.e.* it is not a symmetry of the physical vacuum, the theory contains one massless scalar particle for each broken generator of the original symmetry group.

The Goldstone theorem can be proven as follows. Let us consider a Lagrangian of N_G real scalar fields ϕ_i, belonging to a N_G–dimensional vector Φ,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)(\partial^\mu \Phi) - V(\Phi) .$$

Suppose that G is a continuous group that let the Lagrangian invariant and that Φ transforms like

$$\delta \Phi = -i \; \alpha^a \; T^a \; \Phi .$$

Since the potential is invariant under G, we have

$$\delta V(\Phi) = \frac{\partial V(\Phi)}{\partial \phi_i} \; \delta \phi_i = -i \; \frac{\partial V(\Phi)}{\partial \phi_i} \; \alpha^a \; (T^a)_{ij} \; \phi_j = 0 .$$

The gauge parameters α^a are arbitrary, and we have N_G equations

$$\frac{\partial V(\Phi)}{\partial \phi_i} \; (T^a)_{ij} \; \phi_j = 0 ,$$

for $a = 1, \cdots, N_G$. Taking another derivative of this equation, we obtain

$$\frac{\partial^2 V(\Phi)}{\partial \phi_k \partial \phi_i} \; (T^a)_{ij} \; \phi_j + \frac{\partial V(\Phi)}{\partial \phi_i} \; (T^a)_{ik} = 0 .$$

If we evaluate this result at the vacuum state, $\Phi = \Phi_0$, which minimizes the potential, we get

$$\left.\frac{\partial^2 V(\Phi)}{\partial\phi_k \partial\phi_i}\right|_{\Phi=\Phi_0} (T^a)_{ij}\ \phi_j^0 = 0\ ,$$

or, in terms of the mass matrix,

$$M_{ki}^2\ (T^a)_{ij}\ \phi_j^0 = 0\ . \tag{1.28}$$

If, after we choose a ground state, a sub-group g of G, with dimension n_g, remains a symmetry of the vacuum, then for each generator of g,

$$(T^a)_{ij}\ \phi_j^0 = 0\ \text{ for }\ a = 1, \cdots, n_g \leq N_G\ ,$$

while for the $(N_G - n_g)$ generators that break the symmetry,

$$(T^a)_{ij}\ \phi_j^0 \neq 0\ \text{ for }\ a = n_g + 1, \cdots, N_G\ .$$

Therefore, the relation (1.28) shows that there are $(N_G - n_g)$ zero eigenvalues of the mass matrix: the massless Goldstone bosons.

1.4 The Higgs Mechanism

1.4.1 The Abelian Higgs Mechanism

The Goldstone theorem implies the existence of massless scalar particle(s). However, we do not have any experimental evidence in nature of these particles. In 1964 several authors independently [63, 64, 65] were able to provide a way out to the Goldstone theorem, that is, a field theory with spontaneous symmetry breakdown, but with no massless Goldstone boson(s). The so called Higgs mechanism has an extra bonus: the gauge boson(s) becomes massive. This is accomplished by requiring that the Lagrangian that exhibits the spontaneous symmetry breakdown is also invariant under *local*, rather than global, gauge

transformations. This feature fits very well in the requirements for a gauge theory of electroweak interactions where the short range character of this interaction requires a very massive intermediate particle.

In order to see how this works let us consider again the charged self–interacting scalar Lagrangian (1.24) with the potential (1.25), and let us require a invariance under the *local* phase transformation,

$$\phi \to \exp\left[i\, q\, \alpha(x)\right] \phi \; . \tag{1.29}$$

In order to make the Lagrangian invariant, we introduce a *gauge boson* (A_μ) and the *covariant derivative* (D_μ), following the same principles of Section 1.2

We introduce a *gauge boson* (A_μ) and the *covariant derivative* (D_μ), so that the Lagrangian becomes invariant, following the same principles of Section 1.2

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu + iqA_\mu \quad , \quad \text{with} \quad A_\mu \longrightarrow A'_\mu = A_\mu - \partial_\mu \alpha(x) \; .$$

The spontaneous symmetry breaking occurs for $\mu^2 < 0$, with the vacuum $< |\phi|^2 >$ given by (1.27). There is a very convenient way of parametrizing the new fields, ϕ', that are suitable for small perturbations, *i.e.*,

$$\phi = \exp\left(i\frac{\phi'_2}{v}\right)\frac{(\phi'_1 + v)}{\sqrt{2}} \simeq \frac{1}{\sqrt{2}}\left(\phi'_1 + v + i\phi'_2\right) = \phi' + \frac{v}{\sqrt{2}} \; . \tag{1.30}$$

Therefore the Lagrangian (1.24) becomes,

$$\begin{aligned} \mathcal{L} \;=\; & \frac{1}{2}\partial_\mu\phi'_1\partial^\mu\phi'_1 - \frac{1}{2}(-2\mu^2)\phi'^2_1 + \frac{1}{2}\partial_\mu\phi'_2\partial^\mu\phi'_2 + \text{ interact.} \\ & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}A_\mu A^\mu + qvA_\mu\partial^\mu\phi'_2 \; . \end{aligned} \tag{1.31}$$

This Lagrangian presents a scalar field ϕ'_1 with mass $M_{\phi'_1} = \sqrt{-2\mu^2}$, a massless scalar boson ϕ'_2 (the Goldstone boson) and a massive vector boson A_μ, with mass $M_A = qv$.

However the presence of the last term in (1.31), which is proportional to $A_\mu\partial^\mu\phi'_2$ is quite inconvenient since it mixes the propagators of

A_μ and ϕ_2' particles. In order to eliminate this term, we can choose the gauge parameter in (1.29) to be proportional to ϕ_2' as

$$\alpha(x) = -\frac{1}{qv}\phi_2'(x) \ .$$

In this way, the field ϕ (1.30) becomes,

$$\phi = \exp\left[iq\left(-\frac{\phi_2'}{qv}\right)\right]\exp\left(i\frac{\phi_2'}{v}\right)\frac{(\phi_1'+v)}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\phi_1'+v\right) \ .$$

With this choice of gauge (called unitary gauge) the Goldstone boson disappears, and we get the Lagrangian

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu\phi_1'\partial^\mu\phi_1' - \frac{1}{2}(-2\mu^2)\phi_1'^{\,2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}A_\mu' A^{\mu\,\prime} \\ &\quad + \frac{1}{2}q^2\left(\phi_1'+2v\right)\phi_1' A_\mu' A^{\mu\,\prime} - \frac{\lambda}{4}\phi_1'^{\,3}\left(\phi_1'+4v\right) \ . \end{aligned} \tag{1.32}$$

Where is ϕ_2', the Goldstone boson? To answer this question, it is convenient to count the total number of degrees of freedom from the initial (1.24) and final (1.32) Lagrangians:

Initial $\mathcal{L}$ (1.24)		Final $\mathcal{L}$ (1.32)	
$\phi^{(*)}$ charged scalar :	2	ϕ_1' neutral scalar :	1
A_μ massless vector :	2	A_μ' massive vector :	3
	4		4

As we can see, the corresponding degree of freedom of the Goldstone boson was absorbed by the vector boson that acquires mass. The Goldstone turned into the longitudinal degree of freedom of the vector boson.

1.4.2 The Non–Abelian Case

It is straightforward to generalize the last section's results for a non–Abelian group G of dimension N_G, and generators T^a. In this case, we introduce N_G gauge bosons, such that the covariant derivative is written as

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu - igT^a B^a_\mu \; .$$

After the spontaneous symmetry breaking, a *sub–group* g of dimension n_g remains as a symmetry of the vacuum, that is,

$$T^a_{ij}\, \phi^0_j = 0 \;\; , \quad \text{for} \quad a = 1, \cdots, n_g \; .$$

We would expect the appearance of $(N_G - n_g)$ massless Goldstone bosons. Like in (1.30), we parametrize the original scalar field as

$$\phi = (\tilde{\phi} + v)\; \exp\left(i \frac{\phi^a_{\rm GB} T^a}{v} \right) \;\; ,$$

where T^a are the $(N_G - n_g)$ broken generators that do *not* annihilate the vacuum.

Choose the gauge parameter $\alpha^a(x)$ in order to to eliminate $\phi^a_{\rm GB}$. This will give rise to $(N_G - n_g)$ massive gauge bosons. Counting the total number of degrees of freedom we obtain $N_\phi + 2N_G$, both before and after the spontaneous symmetry breaking:

Before SSB	After SSB
ϕ massless scalar : N_ϕ	$\tilde{\phi}$ massive scalar : $N_\phi - (N_G - n_g)$
B^a_μ massless vector : 2 N_G	$\tilde{B}^a_\mu$ massive vector : 3 $(N_G - n_g)$
	B^a_μ massless vector : 2 n_g

Chapter 2

The Standard Model

2.1 Constructing the Model

2.1.1 General Principles to Construct Gauge Theories

Based on what we have learned from the previous sections, we can establish some quite general principles to construct a gauge theory. The recipe is as follows,

- Choose the gauge group G with N_G generators;
- Add N_G vector fields (gauge bosons) in a specific representation of the gauge group;
- Choose the representation, in general the fundamental representation, for the matter fields (elementary particles);
- Add scalar fields to give mass to (some) vector bosons;
- Define the covariant derivative and write the most general renormalizable Lagrangian, invariant under G, which couples all these fields;

- Shift the scalar fields in such a way that the minimum of the potential is at zero;
- Apply the usual techniques of quantum field theory to verify the renormalizability and to make predictions;
- Check with Nature if the model has anything to do with reality;
- If not, restart from the very beginning!

In fact, there were several attempts to construct a gauge theory for the (electro)weak interaction. In 1957, Schwinger [51] suggested a model based on the group $O(3)$ with a triplet gauge fields (V^+, V^-, V^0). The charged gauge bosons were associated to weak bosons and the neutral V^0 was identified with the photon. This model was proposed before the structure $V - A$ of the weak currents have been established [53, 54, 55].

The first attempt to incorporate the $V - A$ structure in a gauge theory for the weak interactions was made by Bludman [94] in 1958. His model, based on the $SU(2)$ weak isospin group, also required three vector bosons. However in this case the neutral gauge boson was associated to a new massive vector boson that was responsible for weak interactions without exchange of charge (neutral currents). The hypothesis of a neutral vector boson exchanged in weak interaction was also suggested independently by Leite Lopes [56] in the same year. This kind of process was observed experimentally for the first time in 1973 at the CERN neutrino experiment [75].

Glashow [60] in 1961 noticed that in order to accommodate both weak and electromagnetic interactions we should go beyond the $SU(2)$ isospin structure. He suggested the gauge group $SU(2) \otimes U(1)$, where the $U(1)$ was associated to the leptonic hypercharge (Y) that is related to the weak isospin (T) and the electric charge through the analogous of the Gell-Mann–Nishijima formula ($Q = T_3 + Y/2$). The theory now requires four gauge bosons: a triplet (W^1, W^2, W^3) associated to the generators of $SU(2)$ and a neutral field (B) related to $U(1)$. The charged weak bosons appear as a linear combination of W^1 and W^2, while the photon and a neutral weak boson Z^0 are both given by a mixture of W^3 and B. A similar model was proposed by Salam and Ward [67] in 1964.

The mass terms for $W^{\pm}$ and Z^0 were put "by hand". However, as we have seen, this procedure breaks explicitly the gauge invariance of the theory. In 1967, Weinberg [69] and independently Salam [71] in 1968, employed the idea of spontaneous symmetry breaking and the Higgs mechanism to give mass to the weak bosons and, at the same time, to preserve the gauge invariance, making the theory renormalizable as shown later by 't Hooft [73]. The Glashow–Weinberg–Salam model is known, at the present time, as the *Standard Model of Electroweak Interactions*, reflecting its impressive success.

2.1.2 Right– and Left– Handed Fermions

Before the introduction of the Standard Model, let us make an interlude and discuss some properties of the fermionic helicity states. At high energies (*i.e.* for $E \gg m$), the Dirac spinors

$$u(p,s) \quad , \quad \text{and} \quad v(p,s) \equiv C\,\bar{u}^T(p,s) = i\,\gamma_2\,u^*(p,s) \ ,$$

are eigenstates of the γ_5 matrix.

The helicity $+1/2$ (right–handed, R) and helicity $-1/2$ (left–handed, L) states satisfy

$$u_{\substack{\mathrm{R}\\ \mathrm{L}}} = \frac{1}{2}\,(1 \pm \gamma_5)\,u \quad \text{and} \quad v_{\substack{\mathrm{R}\\ \mathrm{L}}} = \frac{1}{2}\,(1 \mp \gamma_5)\,v \ .$$

It is convenient to define the *helicity projectors*:

$$\boxed{L \equiv \frac{1}{2}\,(1 - \gamma_5)} \quad , \quad \boxed{R \equiv \frac{1}{2}\,(1 + \gamma_5)} \ , \tag{2.1}$$

which satisfy the usual properties of projection operators,

$$\begin{aligned} L + R &= 1 \ , \\ R\,L = L\,R &= 0 \ , \\ L^2 &= L \ , \\ R^2 &= R \ . \end{aligned}$$

For the conjugate spinors we have,

$$\begin{aligned}\bar{\psi}_L &= (L\psi)^\dagger\gamma_0 = \psi^\dagger L^\dagger \gamma_0 = \psi^\dagger L\gamma_0 = \psi^\dagger\gamma_0 R = \bar{\psi}R \\ \bar{\psi}_R &= \bar{\psi}L \ .\end{aligned}$$

Let us make some general remarks. First of all, we should notice that fermion mass term mixes right– and left–handed fermion components,

$$\bar{\psi}\psi = \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R \ . \tag{2.2}$$

On the other hand, the electromagnetic (vector) current, does not mix those components, *i.e.*

$$\bar{\psi}\gamma^\mu\psi = \bar{\psi}_R\gamma^\mu\psi_R + \bar{\psi}_L\gamma^\mu\psi_L \ . \tag{2.3}$$

Finally, the $(V-A)$ fermionic weak current can be written in terms of the helicity states as,

$$\bar{\psi}_L\gamma^\mu\psi_L = \bar{\psi}R\gamma^\mu L\psi = \bar{\psi}\gamma^\mu L^2\psi = \bar{\psi}\gamma^\mu L\psi = \frac{1}{2}\,\bar{\psi}\gamma^\mu(1-\gamma_5)\psi \ , \tag{2.4}$$

what shows that only left–handed fermions play a rôle in weak interactions.

2.1.3 Choosing the gauge group

Let us investigate which gauge group would be able to unify the electromagnetic and weak interactions. We start with the charged weak current for leptons. Since electron–type and muon–type lepton numbers are separately conserved, they must form separate representations of the gauge group. Therefore, we refer as ℓ any lepton flavor ($\ell = e,\, \mu,\, \tau$), and the final Lagrangian will be given by a sum over all these flavors.

From Eq. (2.4), we see that the weak current (1.5), for a generic lepton ℓ, is given by,

$$J^+_\mu = \bar{\ell}\gamma_\mu(1-\gamma_5)\nu = 2\,\bar{\ell}_L\gamma_\mu\nu_L \ . \tag{2.5}$$

If we introduce the left–handed isospin doublet ($T = 1/2$),

$$\mathsf{L} \equiv \begin{pmatrix} \nu \\ \ell \end{pmatrix}_L = \begin{pmatrix} L\,\nu \\ L\,\ell \end{pmatrix} = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} , \tag{2.6}$$

where the $T_3 = +1/2$ and $T_3 = -1/2$ components are the left–handed parts of the neutrino and of the charged lepton respectively. Since, there is no right–handed component for the neutrino [*], the right–handed part of the charged lepton is accommodated in a weak isospin singlet ($T = 0$)

$$\mathsf{R} \equiv R\,\ell = \ell_R \; . \tag{2.7}$$

The charged weak current (2.5) can be written in terms of leptonic isospin currents:

$$J^i_\mu = \bar{\mathsf{L}}\,\gamma_\mu\,\frac{\tau^i}{2}\,\mathsf{L}\,,$$

where τ^i are the Pauli matrices. In a explicit form,

$$\begin{aligned}
J^1_\mu &= \frac{1}{2}(\bar{\nu}_L \;\; \bar{\ell}_L)\,\gamma_\mu \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \frac{1}{2}\left(\bar{\ell}_L\gamma_\mu\nu_L + \bar{\nu}_L\gamma_\mu\ell_L\right) \;, \\
J^2_\mu &= \frac{1}{2}(\bar{\nu}_L \;\; \bar{\ell}_L)\,\gamma_\mu \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \frac{i}{2}\left(\bar{\ell}_L\gamma_\mu\nu_L - \bar{\nu}_L\gamma_\mu\ell_L\right) \;, \\
J^3_\mu &= \frac{1}{2}(\bar{\nu}_L \;\; \bar{\ell}_L)\,\gamma_\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \frac{1}{2}\left(\bar{\nu}_L\gamma_\mu\nu_L - \bar{\ell}_L\gamma_\mu\ell_L\right) \;.
\end{aligned}$$

Therefore, the weak charged current (2.5), that couples with intermediate vector boson W^-_μ, can be written in terms of J^1 and J^2 as,

$$J^+_\mu = 2\;\left(J^1_\mu - iJ^2_\mu\right) \;.$$

In order to accommodate the third (neutral) current J^3, we can define the *hypercharge current* by,

$$J^Y_\mu \equiv -\left(\bar{\mathsf{L}}\,\gamma_\mu\,\mathsf{L} + 2\,\bar{\mathsf{R}}\,\gamma_\mu\,\mathsf{R}\right) = -\left(\bar{\nu}_L\gamma_\mu\nu_L + \bar{\ell}_L\gamma_\mu\ell_L + 2\,\bar{\ell}_R\gamma_\mu\ell_R\right) \;.$$

[*] At this moment, we consider that the neutrinos are massless. The possible mass term for the neutrinos will be discussed later, Sec. 2.4.

The *electromagnetic current* can be written as

$$J_\mu^{\rm em} = -\,\bar\ell\,\gamma_\mu\,\ell = -\left(\bar\ell_L\gamma_\mu\ell_L + \bar\ell_R\gamma_\mu\ell_R\right) = J_\mu^3 + \frac{1}{2}J_\mu^Y \ .$$

We should notice that neither T_3 nor Q commute with $T_{1,2}$. However, the 'charges' associated to the currents J^i and J^Y,

$$T^i = \int d^3x\ J_0^i \qquad \text{and} \qquad Y = \int d^3x\ J_0^Y \ ,$$

satisfy the algebra of the $SU(2)\otimes U(1)$ group:

$$[T^i, T^j] = i\ \epsilon^{ijk}T^k \quad , \quad \text{and} \quad [T^i, Y] = 0 \ ,$$

and the Gell-Mann–Nishijima relation between Q and T_3 emerges in a natural way,

$$\boxed{Q = T_3 + \frac{1}{2}Y} \ . \tag{2.8}$$

With the aid of (2.8) we can define the weak hypercharge of the doublet ($Y_{\sf L} = -1$) and of the fermion singlet ($Y_{\sf R} = -2$).

Let us follow our previous recipe for building a general gauge theory. We have just chosen the candidate for the gauge group,

$$\boxed{SU(2)_L \otimes U(1)_Y} \ .$$

The next step is to introduce *gauge fields* corresponding to each generator, that is,

$$\begin{array}{rcl} SU(2)_L & \longrightarrow & W_\mu^1\ ,\ \ W_\mu^2\ ,\ \ W_\mu^3\ , \\ U(1)_Y & \longrightarrow & B_\mu\ . \end{array}$$

Defining the *strength tensors* for the gauge fields according to (1.17) and (1.19),

$$\begin{array}{rcl} W_{\mu\nu}^i & \equiv & \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g\,\epsilon^{ijk}\,W_\mu^j W_\nu^k\ , \\ B_{\mu\nu} & \equiv & \partial_\mu B_\nu - \partial_\nu B_\mu\ , \end{array}$$

we can write the free Lagrangian for the gauge fields following the results (1.18) and (1.20),

$$\mathcal{L}_{\rm gauge} = -\frac{1}{4} W^i_{\mu\nu} W^{i\ \mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \ . \tag{2.9}$$

For the leptons, we write the free Lagrangian,

$$\begin{aligned} \mathcal{L}_{\rm leptons} &= \bar{\mathsf{R}}\, i\, \partial\!\!\!/\, \mathsf{R} + \bar{\mathsf{L}}\, i\, \partial\!\!\!/\, \mathsf{L} \\ &= \bar{\ell}_R\, i\, \partial\!\!\!/\, \ell_R + \bar{\ell}_L\, i\, \partial\!\!\!/\, \ell_L + \bar{\nu}_L\, i\, \partial\!\!\!/\, \nu_L \\ &= \bar{\ell}\, i\, \partial\!\!\!/\, \ell + \bar{\nu}\, i\, \partial\!\!\!/\, \nu \ . \end{aligned} \tag{2.10}$$

Remember that a mass term for the fermions (2.2) mixes the right– and left–components and would break the gauge invariance of the theory from the very beginning.

The next step is to introduce the fermion–gauge boson coupling via the *covariant derivative*, *i.e.*

$$\mathsf{L} \ \ : \quad \partial_\mu + i\, \frac{g}{2}\, \tau^i\, W^i_\mu + i\, \frac{g'}{2}\, Y B_\mu \ , \tag{2.11}$$

$$\mathsf{R} \ \ : \quad \partial_\mu + i\, \frac{g'}{2}\, Y\, B_\mu \ , \tag{2.12}$$

where g and g' are the coupling constant associated to the groups $SU(2)_L$ and $U(1)_Y$ respectively, and

$$Y_{\mathsf{L}_\ell} = -1 \quad , \qquad Y_{\mathsf{R}_\ell} = -2 \ . \tag{2.13}$$

Therefore, the fermion Lagrangian (2.10) becomes

$$\begin{aligned} \mathcal{L}_{\rm leptons} \ \longrightarrow \ & \mathcal{L}_{\rm leptons} + \bar{\mathsf{L}}\, i\gamma^\mu \left(i\frac{g}{2}\tau^i W^i_\mu + i\frac{g'}{2} Y B_\mu \right) \mathsf{L} \\ & + \bar{\mathsf{R}}\, i\gamma^\mu \left(i\frac{g'}{2} Y B_\mu \right) \mathsf{R} \ . \end{aligned} \tag{2.14}$$

Let us first pick up just the "left" piece of (2.14),

$$\mathcal{L}^{\mathsf{L}}_{\rm leptons} = -g\, \bar{\mathsf{L}}\, \gamma^\mu \left(\frac{\tau^1}{2} W^1_\mu + \frac{\tau^2}{2} W^2_\mu \right) \mathsf{L} - g\, \bar{\mathsf{L}}\, \gamma^\mu \frac{\tau^3}{2} \mathsf{L}\, W^3_\mu - \frac{g'}{2} Y \bar{\mathsf{L}}\, \gamma^\mu \mathsf{L}\, B_\mu \ .$$

The first term is *charged* and can be written as

$$\mathcal{L}^{\mathsf{L}(\pm)}_{\rm leptons} = -\frac{g}{2}\,\bar{\mathsf{L}}\,\gamma^\mu \begin{pmatrix} 0 & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & 0 \end{pmatrix} \mathsf{L}\,.$$

This suggests the definition of the *charged gauge bosons* as

$$\boxed{W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp W^2_\mu)}\,, \qquad (2.15)$$

in such a way that

$$\mathcal{L}^{\mathsf{L}(\pm)}_{\rm leptons} = -\frac{g}{2\sqrt{2}}\left[\bar{\nu}\gamma^\mu(1-\gamma_5)\ell\; W^+_\mu + \bar{\ell}\gamma^\mu(1-\gamma_5)\nu\; W^-_\mu\right]\,, \qquad (2.16)$$

reproduces exactly the $(V-A)$ structure of the weak charged current .

When we compare the Lagrangian (2.16) with (1.1) and take into account the result from low–energy phenomenology (1.4) we see that $G_W = g/2\sqrt{2}$ and we obtain the relation

$$\boxed{\frac{g}{2\sqrt{2}} = \left(\frac{M_W^2 G_F}{\sqrt{2}}\right)^{1/2}}\,. \qquad (2.17)$$

Now let us treat the neutral piece of $\mathcal{L}_{\rm leptons}$ (2.14) that contains both left and right fermion components,

$$\begin{aligned}
\mathcal{L}^{(\mathsf{L}+\mathsf{R})(0)}_{\rm leptons} &= -g\,\bar{\mathsf{L}}\left(\gamma^\mu\frac{\tau^3}{2}\right)\mathsf{L}\,W^3_\mu - \frac{g'}{2}\left(\bar{\mathsf{L}}\gamma^\mu Y\mathsf{L} + \bar{\mathsf{R}}\gamma^\mu Y\mathsf{R}\right)B_\mu \\
&= -g\;J^\mu_3\;W^3_\mu - \frac{g'}{2}\;J^\mu_Y\;B_\mu\,, \qquad (2.18)
\end{aligned}$$

where the currents J_3 and J_Y have been defined before,

$$\begin{aligned}
J^\mu_3 &= \frac{1}{2}(\bar{\nu}_L\gamma^\mu\nu_L - \bar{\ell}_L\gamma^\mu\ell_L) \\
J^\mu_Y &= -\left(\bar{\nu}_L\gamma^\mu\nu_L + \bar{\ell}_L\gamma^\mu\ell_L + 2\bar{\ell}_R\gamma^\mu\ell_R\right)\,.
\end{aligned}$$

Note that the ‘charges’ respect the Gell-Mann–Nishijima relation (2.8) and currents satisfy,

$$J_{\rm em} = J_3 + \frac{1}{2}J_Y\,.$$

In order to obtain the right combination of fields that couples to the electromagnetic current, let us make the rotation in the neutral fields, defining the new fields A and Z by,

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix} , \tag{2.19}$$

or,

$$\begin{aligned} W^3_\mu &= \sin\theta_W A_\mu + \cos\theta_W Z_\mu , \\ B_\mu &= \cos\theta_W A_\mu - \sin\theta_W Z_\mu , \end{aligned}$$

where θ_W is called the Weinberg angle and the relation with the $SU(2)$ and $U(1)$ coupling constants hold,

$$\boxed{\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}} \qquad \boxed{\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}} . \tag{2.20}$$

In terms of the new fields, the neutral part of the fermion Lagrangian (2.18) becomes

$$\begin{aligned} \mathcal{L}^{(\mathsf{L+R})(0)}_{\text{leptons}} &= -(g\sin\theta_W J_3^\mu + \frac{1}{2}g'\cos\theta_W J_Y^\mu)A_\mu \\ &\quad +(-g\cos\theta_W J_3^\mu + \frac{1}{2}g'\sin\theta_W J_Y^\mu)Z_\mu \\ &= -g\sin\theta_W\ (\bar{\ell}\gamma^\mu\ell)\ A_\mu \\ &\quad -\frac{g}{2\cos\theta_W}\sum_{\psi_i=\nu,\ell}\bar{\psi}_i\gamma^\mu(g_V^i - g_A^i\gamma_5)\psi_i Z_\mu , \end{aligned} \tag{2.21}$$

and we easily identify the electromagnetic current coupled to the photon field A_μ and the *electromagnetic charge*,

$$\boxed{e = g\sin\theta_W = g'\cos\theta_W} . \tag{2.22}$$

The Standard Model introduces a new ingredient, weak interactions without change of charge, and make a specific prediction for the vector (V) and axial (A) couplings of the Z to the fermions,

$$\boxed{g_V^i \equiv T_3^i - 2Q_i\sin^2\theta_W} , \tag{2.23}$$

$$\boxed{g_A^i \equiv T_3^i} \; . \tag{2.24}$$

This was a very successful prediction of the Standard Model since at that time we had no hint about this new kind of weak interaction. The experimental confirmation of the existence of weak neutral currents occurred more than five years after the model was proposed [75].

Up to now we have in the theory:

- 4 massless gauge fields W_μ^i, B_μ or equivalently, $W_\mu^\pm$, Z_μ, and A_μ;
- 2 massless fermions: ν, ℓ.

The next step will be to add scalar fields in order to break spontaneously the symmetry and use the Higgs mechanism to give mass to the three weak intermediate vector bosons, making sure that the photon remains massless.

2.1.4 The Higgs Mechanism and the W and Z mass

In order to apply the Higgs mechanism to give mass to $W^\pm$ and Z^0, let us introduce the scalar doublet

$$\Phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \; . \tag{2.25}$$

From the relation (2.8), we verify that the hypercharge of the Higgs doublet is $Y = 1$. We introduce the Lagrangian

$$\mathcal{L}_{\text{scalar}} = \partial_\mu \Phi^\dagger \, \partial^\mu \Phi - V(\Phi^\dagger \Phi) \; ,$$

where the potential is given by

$$V(\Phi^\dagger \Phi) = \mu^2 \; \Phi^\dagger \Phi + \lambda \; (\Phi^\dagger \Phi)^2 \; . \tag{2.26}$$

In order to maintain the gauge invariance under the $SU(2)_L \otimes U(1)_Y$, we should introduce the covariant derivative

$$\partial_\mu \to D_\mu = \partial_\mu + i \; g \frac{\tau^i}{2} W_\mu^i + i \; \frac{g'}{2} Y B_\mu \; .$$

We can choose the vacuum expectation value of the Higgs field as,

$$< \Phi >_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} ,$$

where

$$\boxed{v = \sqrt{-\frac{\mu^2}{\lambda}}} . \tag{2.27}$$

Since we want to preserve the exact electromagnetic symmetry to maintain the electric charged conserved, we must break the original symmetry group as

$$SU(2)_L \otimes U(1)_Y \to U(1)_{\rm em} ,$$

i.e. after the spontaneous symmetry breaking, the sub–group $U(1)_{\rm em}$, of dimension 1, should remain as a symmetry of the vacuum.

In this case the corresponding gauge boson, the photon, will remain massless, according to results of section 1.4.2. We can verify that our choice let indeed the vacuum invariant under $U(1)_{\rm em}$. This invariance requires that

$$e^{i\alpha Q} < \Phi >_0 \simeq \ (1 + i\ \alpha\ Q) < \Phi >_0 = < \Phi >_0 ,$$

or, the operator Q annihilates the vacuum, $Q < \Phi >_0 = \ 0$. This is exactly what happens: the electric charge of the vacuum is zero,

$$\begin{aligned} Q < \Phi >_0 &= \left(T_3 + \frac{1}{2} Y \right) < \Phi >_0 \\ &= \frac{1}{2} \left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0 . \end{aligned}$$

The other gauge bosons, corresponding to the *broken generators* T_1, T_2, and $(T_3 - Y/2) = 2T_3 - Q$ should acquire mass. In order to make this explicit, let us parametrize the Higgs doublet *c.f.* (1.30),

$$\begin{aligned} \Phi &\equiv \exp\left(i \frac{\tau^i}{2} \frac{\chi_i}{v} \right) \begin{pmatrix} 0 \\ (v + H)/\sqrt{2} \end{pmatrix} \\ &\simeq < \Phi >_0 + \frac{1}{2\sqrt{2}} \begin{pmatrix} \chi_2 + i\chi_1 \\ 2H - i\chi_3 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i\sqrt{2}\omega^+ \\ v + H - iz^0 \end{pmatrix} . \end{aligned}$$

where $\omega^\pm$ and z^0 are the Goldstone bosons.

Now, if we make a $SU(2)_L$ gauge transformation with $\alpha_i = \chi_i/v$ (unitary gauge) the fields become

$$\Phi \to \Phi' = \exp\left(-i\frac{\tau^i}{2}\frac{\chi_i}{v}\right)\Phi = \frac{(v+H)}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} . \tag{2.28}$$

and the scalar Lagrangian can be written in terms of these new field as

$$\begin{aligned}\mathcal{L}_{\text{scalar}} &= \left|\left(\partial_\mu + ig\frac{\tau^i}{2}W^i_\mu + i\frac{g'}{2}YB_\mu\right)\frac{(v+H)}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}\right|^2 \\ &\quad -\mu^2\,\frac{(v+H)^2}{2} - \lambda\,\frac{(v+H)^4}{4} .\end{aligned} \tag{2.29}$$

In terms of the physical fields $W^\pm$ (2.15) and Z^0 (2.19) the first term of (2.29), that contain the vector bosons, is

$$\begin{aligned}&\left|\begin{pmatrix}0\\ \partial_\mu H/\sqrt{2}\end{pmatrix} + i\frac{g}{2}(v+H)\begin{pmatrix}W^+_\mu\\ (-1/\sqrt{2}c_W)Z_\mu\end{pmatrix}\right|^2 \\ &= \frac{1}{2}\partial_\mu H\partial^\mu H + \frac{g^2}{4}(v+H)^2\left(W^+_\mu W^{-\,\mu} + \frac{1}{2c_W^2}Z_\mu Z^\mu\right) ,\end{aligned} \tag{2.30}$$

where we defined $c_W \equiv \cos\theta_W$.

The quadratic terms in the vector fields, are,

$$\frac{g^2v^2}{4}W^+_\mu W^{-\,\mu} + \frac{g^2v^2}{8\cos^2\theta_W}Z_\mu Z^\mu ,$$

When compared with the usual mass terms for a charged and neutral vector bosons,

$$M_W^2 W^+_\mu W^{-\,\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu ,$$

and we can easily identify

$$\boxed{M_W = \frac{gv}{2}} \qquad \boxed{M_Z = \frac{gv}{2c_W} = \frac{M_W}{c_W}} . \tag{2.31}$$

We can see from (2.30) that no quadratic term in A_μ appears, and therefore, the photon remains massless, as we could expect since the $U(1)_{\rm em}$ remains as a symmetry of the theory.

Taking into account the low–energy phenomenology via the relation (2.17), we obtain for the vacuum expectation value

$$\boxed{v = \left(\sqrt{2}G_F\right)^{1/2} \simeq 246 \text{ GeV}} \,, \tag{2.32}$$

and the Standard Model predictions for the W and Z masses are

$$M_W^2 = \frac{e^2}{4s_W^2}v^2 = \frac{\pi\alpha}{s_W^2}v^2 \simeq \left(\frac{37.2}{s_W}\text{ GeV}\right)^2 \sim (80\text{ GeV})^2 \,,$$

$$M_Z^2 \simeq \left(\frac{37.2}{s_W c_W}\text{ GeV}\right)^2 \sim (90\text{ GeV})^2 \,,$$

where we assumed a experimental value for $s_W^2 \equiv \sin^2\theta_W \sim 0.22$.

We can learn from (2.29) that one scalar boson, out of the four degrees of freedom introduced in (2.25), is remnant of the symmetry breaking. The search for the so called Higgs boson, remains as one of the major challenges of the experimental high energy physics, and will be discussed later in this course (see Sec. 4).

The second term of (2.29) gives rise to terms involving exclusively the scalar field H, namely,

$$-\frac{1}{2}(-2\mu^2)H^2 + \frac{1}{4}\mu^2 v^2\left(\frac{4}{v^3}H^3 + \frac{1}{v^4}H^4 - 1\right) \,. \tag{2.33}$$

In (2.33) we can also identify the Higgs boson mass term with

$$\boxed{M_H = \sqrt{-2\mu^2}} \,, \tag{2.34}$$

and the self–interactions of the H field. In spite of predicting the existence of the Higgs boson, the Standard Model does not give a hint on the value of its mass since μ^2 is *a priori* unknown.

2.2 Some General Remarks

Let us address some general features of the Standard Model:

2.2.1 On the mass matrix of the neutral bosons

In order to have a different view of the rotation (2.19) we analyze the mass term for W^3_μ and B_μ in (2.29). It can be written as

$$\begin{aligned}\mathcal{L}^{W^3-B}_{\text{scalar}} &= \frac{v^2}{2}\left|\left(g\frac{\tau^3}{2}W^3_\mu + \frac{g'}{2}YB_\mu\right)\begin{pmatrix}0\\1\end{pmatrix}\right|^2 \\ &= \frac{v^2}{8}\left[\begin{pmatrix}B_\mu & W^3_\mu\end{pmatrix}\begin{pmatrix}g'^2 & -gg'\\ -gg' & g^2\end{pmatrix}\begin{pmatrix}B^\mu\\ W^{3\,\mu}\end{pmatrix}\right] .\end{aligned}$$

The mass matrix is not diagonal and has two eigenvalues, namely,

$$0 \quad \text{and} \quad \left(\frac{1}{2}\right)\frac{(g^2+g'^2)v^2}{4} = \frac{1}{2}M_Z^2 ,$$

which correspond exactly to the photon ($M_A = 0$) and Z mass (2.31).

We obtain a better understanding of the meaning of the Weinberg angle rotation by noticing that the same rotation matrix used to define the physical fields in (2.19),

$$R_W = \begin{pmatrix}\cos\theta_W & \sin\theta_W\\ -\sin\theta_W & \cos\theta_W\end{pmatrix} ,$$

is the one that diagonalizes the mass matrix for the neutral gauge bosons, *i.e.*

$$R_W\,\frac{v^2}{4}\begin{pmatrix}g'^2 & -gg'\\ -gg' & g^2\end{pmatrix}R_W^T = \begin{pmatrix}0 & 0\\ 0 & M_Z^2\end{pmatrix} .$$

2.2.2 On the ρ Parameter

We can define a dimensionless parameter ρ by:

$$\rho = \frac{M_W^2}{\cos^2\theta_W M_Z^2} ,$$

that represents the relative strength of the neutral and charged effective Lagrangians ($J^{0\,\mu} J^0_\mu / J^{+\,\mu} J^-_\mu$),

$$\rho = \frac{g^2}{8\cos^2\theta_W M_Z^2} \Bigg/ \frac{g^2}{8 M_W^2} \quad .$$

In the Standard Model, at tree level, the ρ parameter is 1. This is not a general consequence of the gauge invariance of the model, but it is, in fact, a successful prediction of the model.

In a model with an arbitrary number of Higgs multiplets ϕ_i with isospin T_i and third component T_i^3, and vacuum expectation value v_i, the ρ parameter is given by

$$\rho = \frac{\sum_i \left[T_i(T_i+1) - (T_{3\,i})^2\right] v_i^2}{2\sum_i (T_{3\,i})^2 v_i^2} \ ,$$

which is 1 for an arbitrary number of doublets.

Therefore, ρ represents a good test for the isospin structure of the Higgs sector. As we will see later, it is also sensitive to radiative corrections.

2.2.3 On the Gauge Fixing Term

The unitary gauge chosen in (2.28) has the great advantage of making the physical spectrum clear: the $W^\pm$ and Z^0 become massive and no massless Goldstone boson appears in the spectrum.

In this gauge the vector boson (V) propagator is given by

$$P^U_{\mu\nu}(V) = \frac{-i}{q^2 - M_V^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_V^2}\right) \ .$$

Notice that $P^U_{\mu\nu}$ does *not* go like $\sim 1/q^2$ as $q \to \infty$ due to the term proportional to $q_\mu q_\nu$. This feature has some very unpleasant consequences. First of all there are very complicated cancellations in the invariant amplitudes involving the vector boson propagation at high energies.

More dramatic is the fact that it is very hard to prove the renormalizability of the theory since it makes use of power counting analysis in the loop diagrams.

A way out to this problem [73, 95] is to add a *gauge–fixing* term to the original Lagrangian,

$$\mathcal{L}_{\rm gf} = -\frac{1}{2}\left(2G_W^+G_W^- + G_Z^{\,2} + G_A^{\,2}\right) \ ,$$

with

$$\begin{aligned}
G_W^\pm &= \frac{1}{\sqrt{\xi_W}}\left(\partial^\mu W_\mu^\pm \mp i\xi_W M_W \omega^\pm\right) \ , \\
G_Z &= \frac{1}{\sqrt{\xi_Z}}\left(\partial^\mu Z_\mu - \xi_Z M_Z z\right) \ , \\
G_A &= \frac{1}{\sqrt{\xi_A}}\partial^\mu A_\mu \ ,
\end{aligned}$$

where $\omega^\pm$ and z are the Goldstone bosons. This is called the R$_\xi$ gauge.

Notice, for instance, that,

$$\begin{aligned}
-\frac{1}{2}G_Z^{\,2} &= -\frac{1}{2\xi_Z}(\partial_\mu Z^\mu - \xi_Z M_Z z)^2 \\
&= \frac{1}{2}Z_\mu\left(\frac{1}{\xi_Z}\partial^\mu\partial^\nu\right)Z_\nu - \frac{1}{2}\,\xi_Z\,M_Z^2\,z^2 + M_Z\,z\,\partial^\mu Z_\mu \ ,
\end{aligned}$$

where the last term that mixes the Goldstone (z) and the vector boson ($\partial^\mu Z_\mu$) is canceled by an identical term that comes from the scalar Lagrangian [see Eq. (1.31)].

In the R$_\xi$ gauge the vector boson propagators is

$$P_{\mu\nu}^{R_\xi}(V) = \frac{-i}{q^2 - M_V^2}\left[g_{\mu\nu} - (1-\xi_V)\frac{q_\mu q_\nu}{q^2 - \xi_V M_V^2}\right] \ . \tag{2.35}$$

In this gauge the Goldstone bosons, with mass $\sqrt{\xi_V}M_V$, remain in the spectrum and their propagators are given by,

$$P^{R_\xi}(GB) = \frac{i}{q^2 - \xi_V M_V^2} \ .$$

and the physical Higgs propagator remains the same.

In the limit of $\xi_V \to \infty$ the Goldstone bosons disappear and the unitary gauge is recovered. Other gauge choices like Landau gauge ($\xi_V \to 0$) and Feynman gauge ($\xi_V \to 1$) are contained in (2.35). Therefore, all physical processes should not depend on the parameter ξ_V.

2.2.4 On the Measurement of $\sin^2\theta_W$ at Low Energies

The value of the Weinberg angle is not predicted by the Standard Model and should be extract from the experimental data. Once we have measured θ_W (and of course, e) the value of the $SU(2)_L$ and $U(1)_Y$ coupling constants are determined via (2.22).

At low energies the value of $\sin^2\theta_W$ can be obtained from different reactions. For instance:

• The cross section for elastic neutrino–lepton scattering

$$\begin{matrix}\nu_\mu \\ \bar{\nu}_\mu\end{matrix} + e \;\to\; \begin{matrix}\nu_\mu \\ \bar{\nu}_\mu\end{matrix} + e \;,$$

which involve a t–channel Z^0 exchange is given by

$$\sigma = \frac{G_F^2 M_e E_\nu}{2\pi}\left[(g_V^e \pm g_A^e)^2 + \frac{1}{3}\,(g_V^e \mp g_A^e)^2\right] \;.$$

The vector and axial couplings of the electron to the Z are given by (2.23) and (2.24),

$$g_V^e = -\frac{1}{2} + 2\sin^2\theta_W \quad , \qquad g_A^e = -\frac{1}{2} \;,$$

and depend on the $\sin^2\theta_W$. For ν_e reaction we should make the substitution $g_{V,A}^e \to (g_{V,A}^e + 1)$ since in this case there is also a W exchange contribution. When the ratio $\sigma(\nu_\mu e)/\sigma(\bar{\nu}_\mu e)$ is measured the systematic uncertainties cancel out and yields $\sin^2\theta_W = 0.221 \pm 0.008$ [32].

• Deep inelastic neutrino scattering from isoscalar targets (N). The ratio between the neutral (NC) and charged (CC) current cross sections

$$R_{\nu(\bar{\nu})} \equiv \frac{\sigma^{\rm NC}[\nu(\bar{\nu})N]}{\sigma^{\rm CC}[\nu(\bar{\nu})N]} \;,$$

depends on the $\sin^2\theta_W$ as

$$R_{\nu(\bar{\nu})} \simeq \frac{1}{2} - \sin^2\theta_W + \frac{5}{9}[1 + r(1/r)]\sin^4\theta_W \ ,$$

with $r \equiv \sigma^{\rm CC}(\bar{\nu}N)/\sigma^{\rm CC}(\nu N) \simeq 0.44$. The measurement of these reactions yields $\sin^2\theta_W = 0.226 \pm 0.004$ [32].

• Atomic parity violation. The Z^0 mediated electron–nucleus interaction in cesium, thallium, lead and bismuth can be described by the interaction Hamiltonian,

$$\mathcal{H} = \frac{G_F}{2\sqrt{2}}\, Q_W\, \gamma_5\, \rho_{\rm nuc} \ ,$$

with Q_W being the "weak charge" that depends on the Weinberg angle,

$$Q_W \simeq Z(1 - 4\sin^2\theta_W) - N \ ,$$

where $Z(N)$ is the number of protons (neutrons). This measurement furnishes $\sin^2\theta_W = 0.220 \pm 0.003$ [32].

Nevertheless, the most precise measurements of the Weinberg angle are obtained at high energies, for instance in electron–positron collisions at the Z pole (see section 3.2).

2.2.5 On the Lepton Mass

Note that the charged lepton is still massless, since

$$M_\ell\, \bar{\ell}\, \ell = M_\ell\, (\bar{\ell}_R\, \ell_L + \bar{\ell}_L\, \ell_R) \ ,$$

mixes L and R components and breaks gauge invariance. A way to give mass in a gauge invariant way is via the Yukawa coupling of the leptons with the Higgs field (2.28), that is,

$$\begin{aligned}
\mathcal{L}^\ell_{\rm yuk} &= -G_\ell\left[\bar{\mathsf{R}}\,\left(\Phi^\dagger\,\mathsf{L}\right) + \left(\bar{\mathsf{L}}\,\Phi\right)\,\mathsf{R}\right] \\
&= -G_\ell\frac{(v+H)}{\sqrt{2}}\left[\bar{\ell}_R\,(0 \quad 1)\begin{pmatrix}\nu_L \\ \ell_L\end{pmatrix} + (\bar{\nu_L} \quad \bar{\ell_L})\begin{pmatrix}0 \\ 1\end{pmatrix}\ell_R\right] \\
&= -\frac{G_\ell\, v}{\sqrt{2}}\,\bar{\ell}\,\ell - \frac{G_\ell}{\sqrt{2}}\,\bar{\ell}\,\ell\, H \ . \qquad (2.36)
\end{aligned}$$

Thus, we can identify the charged lepton mass,

$$\boxed{M_\ell = \frac{G_\ell \, v}{\sqrt{2}}} \,. \tag{2.37}$$

We notice that this procedure is able to generate a mass term for the fermion in a gauge invariant way. However, it does not specify the value of the mass since the Yukawa constant G_ℓ introduced in (2.36) is arbitrary.

As a consequence, we obtain the Higgs–lepton coupling with strength,

$$\boxed{C_{\bar{\ell}\ell H} = \frac{M_\ell}{v}} \,, \tag{2.38}$$

which is a precise prediction of the Standard Model that should also be checked experimentally.

2.2.6 On the Cross Sections $e^+e^- \to W^+W^-$

A very interesting example on how the Standard Model is able to improve the unitarity behavior of the cross sections is provided by the $e^+e^- \to W^+W^-$ processes, which is presented in Fig. 5.

The first two diagrams are the t–channel neutrino exchange, similar to the contribution of Fig. 1, and the s–channel photon exchange. Both of them are present in any theory containing charged intermediate vector boson. However, the Standard Model introduces two new contributions: the neutral current contribution (Z exchange) and the Higgs boson exchange (H).

The leading p–wave divergence of the neutrino diagram, which is proportional to s, is analogous to the one found in the reaction $\nu\bar{\nu} \to W^+W^-$. However, in this case it is exactly canceled by the sum of the contributions of the photon (A) and the Z. This delicate canceling is a direct consequence of the gauge structure of the theory [96].

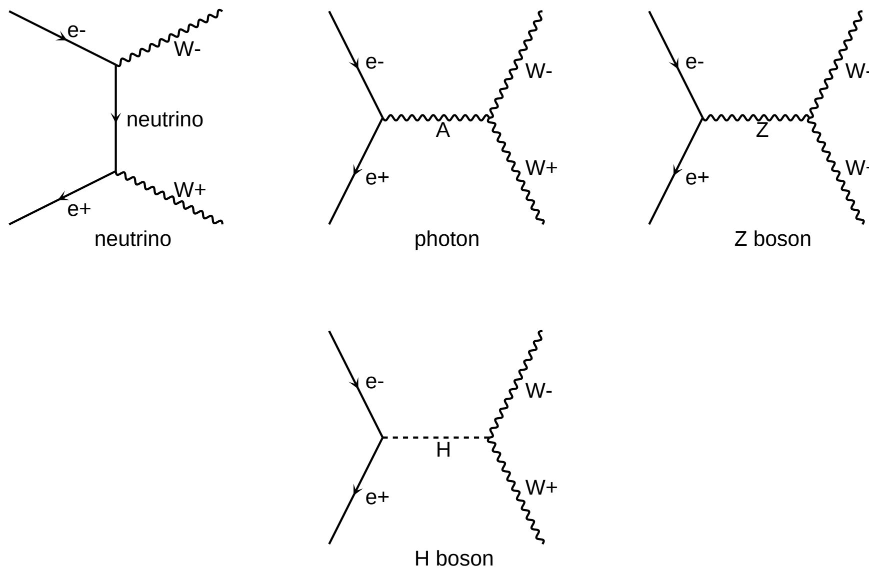


Fig. 5: Feynman diagram for the reaction $e^+e^- \rightarrow W^+W^-$.

However, the s–wave scattering amplitude is proportional to $(m_f\sqrt{s})$ and, therefore, is also divergent at high energies. This remaining divergence is canceled by the Higgs exchange diagram. Therefore, the existence of a scalar boson, that gives rise to a s–wave contribution and couples proportionally to the fermion mass, is an essential ingredient of the theory. In Quigg's words [1],

> *"If the Higgs boson did not exist, we should have to invent something very much like it."*

2.3 Introducing the Quarks

In order to introduce the strong interacting particles in the Standard Model we shall first examine what happens with the hadronic neutral current when the Cabibbo angle (1.7) is taken into account. We can write the hadronic neutral current in terms of the quarks u and d',

$$\begin{aligned} J^H_\mu(0) &= \bar{u}\gamma_\mu(1-\gamma_5)u + \bar{d}'\gamma_\mu(1-\gamma_5)d' \\ &= \bar{u}\gamma_\mu(1-\gamma_5)u + \cos^2\theta_C\ \bar{d}\gamma_\mu(1-\gamma_5)d + \sin^2\theta_C\ \bar{s}\gamma_\mu(1-\gamma_5)s \\ &+ \cos\theta_C\sin\theta_C\left[\bar{d}\gamma_\mu(1-\gamma_5)s + \bar{s}\gamma_\mu(1-\gamma_5)d\right] . \end{aligned}$$

We should notice that the last term generates flavor changing neutral currents (FCNC), *i.e.* transitions like $d+\bar{s} \leftrightarrow \bar{d}+s$, with the same strength of the usual weak interaction. However, the observed FCNC processes are extremely small. For instance, the branching ratio of charged kaons decaying via charged current is,

$$BR\left(K^+_{u\bar{s}} \to W^+ \to \mu^+\nu\right) \simeq 63.5\% \ ,$$

while process involving FCNC are very small [32]:

$$\begin{aligned} BR\left(K^+_{u\bar{s}} \to \pi^+_{u\bar{d}}\nu\bar{\nu}\right) &\simeq 4.2\times 10^{-10} \ , \\ BR\left(K^L_{d\bar{s}} \to \mu^+\mu^-\right) &\simeq 7.2\times 10^{-9} \ . \end{aligned}$$

In 1970, Glashow, Iliopoulos, and Maiani proposed the GIM mechanism [72]. They consider a fourth quark flavor, the charm (c), already introduced by Bjorken and Glashow in 1963. This extra quark completes the symmetry between quarks (u, d, c, and s) and leptons (ν_e, e, ν_μ, and μ) and suggests the introduction of the weak doublets

$$\begin{aligned} \mathsf{L}_U &\equiv \begin{pmatrix} u \\ d' \end{pmatrix}_L = \begin{pmatrix} u \\ \cos\theta_C\ d + \sin\theta_C\ s \end{pmatrix}_L , \\ \mathsf{L}_C &\equiv \begin{pmatrix} c \\ s' \end{pmatrix}_L = \begin{pmatrix} c \\ -\sin\theta_C\ d + \cos\theta_C\ s \end{pmatrix}_L . \end{aligned} \tag{2.39}$$

and the right–handed quark singlets,

$$\mathsf{R}_U \ , \quad \mathsf{R}_D \ , \quad \mathsf{R}_S \ , \quad \mathsf{R}_C \ . \tag{2.40}$$

Notice that now all particles, *i.e.* the $T_3 = \pm 1/2$ fields, have also the right components to enable a mass term for them.

In order to introduce the quarks in the Standard Model, we should start, just like in the leptonic case (2.10), from the free massless Dirac Lagrangian for the quarks,

$$\begin{aligned} \mathcal{L}_{\rm quarks} &= \bar{\mathsf{L}}_U \, i \, \partial\!\!\!/ \, \mathsf{L}_U + \bar{\mathsf{L}}_C \, i \, \partial\!\!\!/ \, \mathsf{L}_C \\ &\quad + \bar{\mathsf{R}}_U \, i \, \partial\!\!\!/ \, \mathsf{R}_U + \cdots + \bar{\mathsf{R}}_C \, i \, \partial\!\!\!/ \, \mathsf{R}_C \, . \end{aligned} \tag{2.41}$$

We should now introduce the gauge bosons interaction via the covariant derivatives (2.11) with the quark hypercharges determined by the Gell-Mann–Nishijima relation (2.8), in such a way that the up–type quark charge is $+2/3$ and the down–type $-1/3$,

$$Y_{\mathsf{L}_Q} = \frac{1}{3} \quad , \quad Y_{\mathsf{R}_U} = \frac{4}{3} \quad , \quad Y_{\mathsf{R}_D} = -\frac{2}{3} \, . \tag{2.42}$$

Therefore, the charged weak couplings quark–gauge bosons, is given by,

$$\mathcal{L}^{(\pm)}_{\rm quarks} = \frac{g}{2\sqrt{2}} \left[\bar{u}\gamma^\mu (1-\gamma_5) d' + \bar{c}\gamma^\mu (1-\gamma_5) s' \right] W^+_\mu + \text{h.c.} \, . \tag{2.43}$$

On the other hand, the neutral current receives a new contribution proportional to

$$\bar{c}\gamma_\mu (1-\gamma_5) c + \bar{s}'\gamma_\mu (1-\gamma_5) s'$$

and becomes diagonal in the quarks flavors, since the inconvenient terms of $J^H_\mu(0)$ cancels out, avoiding the phenomenological problem with the FCNC. For instance, for the process $K^L \to \mu^+\mu^-$, the GIM mechanism introduces a new box contribution containing the c–quark that cancels most of the u–box contribution and gives a result in agreement with experiment [97].

Finally, the neutral current interaction of the quarks become,

$$\mathcal{L}^{(0)}_{\rm quarks} = -\frac{g}{2c_W} \sum_{\psi_q = u, \cdots, c} \bar{\psi}_q \gamma^\mu (g^q_V - g^q_A \gamma_5) \psi_q \, Z_\mu \, , \tag{2.44}$$

with the vector and axial couplings for the quarks given by (2.23) and (2.24), for $i = q$.

2.3.1 On Anomaly Cancellation

In field theory, some loop corrections can violate a classical local conservation law, derived from gauge invariance via Noether's theorem The so–called anomaly is a disaster since it breaks Ward–Takahashi identities and invalidate the proofs of renormalizability. The vanishing of the anomalies is so important that have been used as a guide for constructing realistic theories.

Let us consider a generic theory with Lagrangian

$$\mathcal{L}_{\rm int} = -g \left(\bar{R}\, \gamma^\mu\, T^a_+\, R + \bar{L}\, \gamma^\mu\, T^a_-\, L \right)\, \mathcal{V}^a_\mu \ ,$$

where $T^a_\pm$ are the generators in the right $(+)$ and left $(-)$ representation of the matter fields, and $\mathcal{V}^a_\mu$ are the gauge bosons. This theory will be anomaly free if

$$\mathcal{A}^{abc} = \mathcal{A}^{abc}_+ - \mathcal{A}^{abc}_- = 0 \ ,$$

where $\mathcal{A}^{abc}_\pm$ is given by the following trace of generators

$$\mathcal{A}^{abc}_\pm \equiv \mathbf{Tr}\left[\{T^a_\pm, T^b_\pm\} T^c_\pm \right] \ .$$

In a $V - A$ gauge theory like the Standard Model, the only possible anomalies come from VVA triangle loops, *i.e.* loops with two vectors and one axial vertex and are proportional to:

$$\begin{aligned} SU(2)^2U(1) \quad &: \quad \mathbf{Tr}\left[\{\tau^a, \tau^b\} Y \right] = \mathbf{Tr}\left[\{\tau^a, \tau^b\} \right] \mathbf{Tr}\left[Y \right] \propto \sum_{doub.} Y \\ U(1)^3 \quad &: \quad \mathbf{Tr}\left[Y^3 \right] \propto \sum_{ferm.} Y^3 \ . \end{aligned}$$

Remembering the value of the hypercharge of the leptons (2.13) and quarks (2.42), we can write for the $SU(2)^2U(1)$ case,

$$\mathcal{A}^{abc} \propto -\sum_{doub.} Y = -\left[-1 + 3\left(\frac{1}{3} \right) \right] = 0 \ ,$$

and for the $U(1)^3$ case,

$$\begin{aligned}\mathcal{A}^{abc} \propto \sum_{ferm} Y_+^3 - Y_-^3 &= \left\{(-2)^3 + 3\left[\left(\frac{4}{3}\right)^3 + \left(\frac{-2}{3}\right)^3\right]\right\} \\ &- \left\{(-1)^3 + (-1)^3 + 3\left[\left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^3\right]\right\} = 0 \ .\end{aligned}$$

where the 3 colors of the quarks were taken into account.

This shows that the Standard Model is free from anomalies if the fermions appears in complete multiplets, with the general structure:

$$\left\{\left(\begin{array}{c}\nu_e \\ e\end{array}\right)_L ,\ e_R\ ,\ \left(\begin{array}{c}u \\ d\end{array}\right)_L ,\ u_R\ ,\ d_R\right\}\ ,$$

that should be repeated always respecting this same structure:

$$\left\{\left(\begin{array}{c}\nu_\mu \\ \mu\end{array}\right)_L ,\ \mu_R\ ,\ \left(\begin{array}{c}c \\ s\end{array}\right)_L ,\ c_R\ ,\ s_R\right\}\ ,$$

The discovery of the τ lepton in 1975 [77], and of a fifth quark flavor, the b [78], two years later, were the evidence for a third fermion generation,

$$\left\{\left(\begin{array}{c}\nu_\tau \\ \tau\end{array}\right)_L ,\ \tau_R\ ,\ \left(\begin{array}{c}t \\ b\end{array}\right)_L ,\ t_R\ ,\ b_R\right\}\ .$$

The existence of complete *generations*, with no missing partner, is essential for the vanishing of anomalies. This was a compelling theoretical argument in favor of the existence of a top quark before its discovery in 1995 [87, 88].

2.3.2 The Quark Masses

In order to generate mass for both the up ($U_i = u$, c, and t) and down ($D_i = d$, s, and b) quarks, we need a $Y = -1$ Higgs doublet. Defining the conjugate doublet Higgs as,

$$\tilde{\Phi} = i\ \sigma_2\ \Phi^* = \left(\begin{array}{c}\phi^{0^*} \\ -\phi^-\end{array}\right)\ , \tag{2.45}$$

we can write the Yukawa Lagrangian for three generations of quarks as,

$$\mathcal{L}^q_{\text{yuk}} = -\sum_{i,j=1}^{3} \left[G^U_{ij}\, \bar{\mathsf{R}}_{U_i} \left(\tilde{\Phi}^\dagger \mathsf{L}_j \right) + G^D_{ij}\, \bar{\mathsf{R}}_{D_i} \left(\Phi^\dagger \mathsf{L}_j \right) \right] + \text{h.c.}\ . \tag{2.46}$$

From the vacuum expectation values of Φ and $\tilde{\Phi}$ doublets, we obtain the mass terms for the up

$$\overline{(u',\ c',\ t')_R}\, \mathcal{M}^U \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}_L + \text{ h.c.}\ ,$$

and down quarks

$$\overline{(d',\ s',\ b')_R}\, \mathcal{M}^D \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L + \text{ h.c.}\ ,$$

with the non–diagonal matrices $\mathcal{M}^{U(D)}_{ij} = (v/\sqrt{2})\, G^{U(D)}_{ij}$.

The weak eigenstates (q') are linear superposition of the mass eigenstates (q) given by the unitary transformations:

$$\begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}_{L,R} = U_{L,R} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R}\ , \qquad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_{L,R} = D_{L,R} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R}\ ,$$

where $U(D)_{L,R}$ are unitary matrices to preserve the form of the kinetic terms of the quarks (2.41). These matrices diagonalize the mass matrices, *i.e.*,

$$\begin{aligned} U_R^{-1} \mathcal{M}^U U_L &= \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \\ D_R^{-1} \mathcal{M}^D D_L &= \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}\ . \end{aligned}$$

The $(V-A)$ charged weak current (2.43), for three generations, will be proportional to

$$\overline{(u',\ c',\ t')_L}\,\gamma_\mu \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = \overline{(u,\ c,\ t)_L}\ \ (U_L^\dagger D_L)\,\gamma_\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L ,$$

with the generation mixing of the mass eigenstates (q) described by:

$$V \equiv (U_L^\dagger\ D_L) .$$

On the other hand, for the neutral current of the quarks (2.44), now becomes,

$$\overline{(u',\ c',\ t')_L}\,\gamma_\mu \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}_L = \overline{(u,\ c,\ t)_L}\ \ (U_L^\dagger U_L)\,\gamma_\mu \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L .$$

We can notice that there is no mixing in the neutral sector (FCNC) since the matrix U_L is unitary: $U_L^\dagger U_L = 1$.

The quark mixing, by convention, is restricted to the down quarks, that is with $T_3^q = -1/2$,

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L .$$

V is the Cabibbo–Kobayashi–Maskawa matrix [61, 74], that can be parametrized as

$$V = R_1(\theta_{23})R_2(\theta_{13},\delta_{13})R_3(\theta_{12}) ,$$

where $R_i(\theta_{jk})$ are rotation matrices around the axis i, the angle θ_{jk} describes the mixing of the generations j and k and δ_{13} is a phase.

We should notice that, for three generations, it is not always possible to choose the V matrix to be real, that is $\delta_{13} = 0$, and therefore the weak interaction can violate CP and T [†].

[†]The violation of CP can also occur in the interaction of scalar bosons, when we have two or more scalar doublets. For a review see Ref. [98].

The Cabibbo–Kobayashi–Maskawa matrix can be written as

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\ e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\ e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}\ e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\ e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\ e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} ,$$

where $s_{ij}(c_{ij}) \equiv \sin(\cos)\theta_i j$. Notice that, in the limit of $\theta_{23} = \theta_{13} \to 0$, we associate $\theta_{12} \to \theta_C$, the Cabibbo angle (1.7), and

$$V \to \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

Using unitarity constraints and assuming only three generations the experimental value for the elements of the matrix V, with 90% of C.L., can be extract from weak quark decays and from deep inelastic neutrino scattering [32],

$$V = \begin{pmatrix} 0.9742 - 0.9757 & 0.219 - 0226 & 0.002 - 0.005 \\ 0.219 - 0.225 & 0.9734 - 09749 & 0.037 - 0.043 \\ 0.004 - 0.014 & 0.035 - 0.043 & 0.9990 - 0.9993 \end{pmatrix} .$$

2.4 The Standard Model Lagrangian

We end this chapter giving a birds' eye view of the Standard Model, putting all terms together and writing the whole Lagrangian in a schematic way.

Gauge–boson + Scalar

The gauge–boson (2.9) and the scalar (2.29) Lagrangians give rise to the free Lagrangian for the photon, W, Z, and the Higgs boson. Besides that, they generate triple and quartic couplings among the vector

bosons and also couplings involving the Higgs boson:

$$
\begin{aligned}
& \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{scalar}} = \\
- & \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}W^{+}_{\mu\nu}W^{-\,\mu\nu} + M_W^2 W^{+}_{\mu}W^{-\,\mu} \\
- & \frac{1}{4}Z_{\mu\nu}Z^{\,\mu\nu} + M_Z^2 Z_\mu Z^\mu + \frac{1}{2}\partial_\mu H \partial^\mu H - \frac{1}{2}M_H^2 H^2 \\
+ & \boxed{W^+W^-A} + \boxed{W^+W^-Z} \\
+ & \boxed{W^+W^-AA} + \boxed{W^+W^-ZZ} + \boxed{W^+W^-AZ} + \boxed{W^+W^-W^+W^-} \\
+ & \boxed{HHH} + \boxed{HHHH} \\
+ & \boxed{W^+W^-H} + \boxed{W^+W^-HH} + \boxed{ZZH} + \boxed{ZZHH}\,.
\end{aligned}
\tag{2.47}
$$

The vector–boson self–couplings that appear in (2.47) are strictly constrained by the $SU(2)_L \otimes U(1)_Y$ gauge invariance and any small deviation from the Standard Model predictions would destroy, for instance, the precise cancellation of the high–energy behavior between the various diagrams, giving rise to an anomalous growth of the cross section with energy. Therefore, the careful study of the vector–boson self–interactions is a important test of the Standard Model (see M. E. Pol, these Proceedings).

Leptons + Yukawa

The leptonic (2.14) and the Yukawa (2.36) Lagrangians are responsible for the lepton free Lagrangian and for the couplings with the gauge bosons: photon (QED interaction), W (charged weak current) and Z (neutral weak current). The mass terms are generated by the Yukawa interaction which also gives rise to the coupling of the massive lepton with the Higgs boson:

$$
\begin{aligned}
& \mathcal{L}_{\text{leptons}} + \mathcal{L}^{\ell}_{\text{yuk}} = \\
& \sum_{\ell=e,\mu,\tau} \bar{\ell}(i\,\partial\!\!\!/ - m_\ell)\ell + \sum_{\nu_\ell=\nu_e,\nu_\mu,\nu_\tau} \bar{\nu}_\ell(i\,\partial\!\!\!/)\nu_\ell \\
+ & \boxed{\bar{\ell}\,\ell\,A} + \boxed{\bar{\nu}_\ell\,\ell\,W^+} + \boxed{\bar{\ell}\,\nu_\ell\,W^-} + \boxed{\bar{\ell}\,\ell\,Z} + \boxed{\bar{\nu}_\ell\,\nu_\ell\,Z} \\
+ & \boxed{\bar{\ell}\,\ell\,H}\,.
\end{aligned}
\tag{2.48}
$$

Even if the neutrinos have mass, as seems to suggest the recent experimental results [99, 100], their Dirac mass terms could be incorporated in the framework of the Standard Model without any difficulty. The procedure would be similar to the one that lead to the quark mass terms, that is, introducing a right–handed component of the neutrino and a Yukawa coupling with the conjugate Higgs doublet (2.45). One may notice, however, than being electrically neutral neutrinos may also have a Majorana mass which violates lepton number. The simultaneous existence of both type of mass terms, Dirac and Majorana, can be used to explain the smallness of the neutrino mass as compared to the charged leptons via the so called see-saw mechanism [101].

Quarks + Yukawa

The quark Lagrangian (2.41) and the corresponding Yukawa interaction (2.46) give rise to the free Dirac term and to the electromagnetic and weak interaction of the quarks. A quark–Higgs coupling is also generated,

$$\begin{aligned} & \mathcal{L}_{\text{quarks}} + \mathcal{L}^{q}_{\text{Yuk}} = \\ & \sum_{q=u,\cdots,t} \bar{q}(i \not{\partial} - m_q) q \\ + \; & \boxed{\bar{q}\, q\, A} \\ + \; & \boxed{\bar{u}\, d'\, W^+} + \boxed{\bar{d}'\, u\, W^-} + \boxed{\bar{q}\, q\, Z} \\ + \; & \boxed{\bar{q}\, q\, H}\,. \end{aligned} \tag{2.49}$$

Besides the propagators and couplings presented above, in a general R_ξ gauge, we should also take into account the contribution of the Goldstone bosons and of the ghosts. The Faddeev–Popov ghosts [70] are important to cancel the contribution of the unphysical (timelike and longitudinal) degrees of freedom of the gauge bosons.

A practical guide to derive the Feynman rules for the vertex and propagators can be found, for instance, in Ref. [2], where the complete set of rules for the Standard Model is presented.

Chapter 3

Beyond the Trees

3.1 Radiative Corrections to the Standard Model

It was shown that the Standard Model is a renormalizable field theory. This means that when we go beyond the tree level (Born approximation) we are still able to make definite predictions for observables. The general procedure to evaluate these quantities at the quantum level is to collect and evaluate all the loop diagrams up to a certain level. Many of these contributions are ultraviolet divergent and a convenient regularization method (*e.g.* dimensional regularization) should be used to isolate the divergent parts. These divergences are absorbed in the bare couplings and masses of the theory. Assuming a renormalization condition (*e.g.* on–shell or $\overline{\mathrm{MS}}$ scheme), we can evaluate all the counterterms. After all these ingredients are put together we are able to obtain finite results for S–matrix elements that can be translated in, for instance, cross sections and decay widths. The predictions of the Standard Model for several observables are obtained and can be compared with the experimental results for these quantities enabling the theory to be falsified (in the Popperian sense).

The subject of renormalization is very cumbersome and deserves a whole course by itself. Here we want to give just the minimum necessary tools to enable the reader to appreciate the astonishing agreement

of the Standard Model, even at the quantum level, with the recent experimental results. Very good accounts of the electroweak radiative corrections can be found elsewhere [102, 103, 104].

Let us start considering the Standard Model Lagrangian which is given by the sum of the contributions (2.47), (2.48), and (2.49). The $\mathcal{L}_{SM}$ is a function of coupling constants g and g' and of the vacuum expectation value of the Higgs field, v. The observables can be determined in terms of these parameters and any possible dependence on other quantities like M_H or m_t appears just through radiative corrections.

Therefore, we need three precisely known observables in order to determine the basic input parameters of the model. A natural choice will be the most well measured quantities, like, *e.g.*:

- The electromagnetic fine structure constant that can be extracted, for instance, from the electron $g_e - 2$ or from the quantum Hall effect,

$$\alpha(0) = 1/137.0359895(61) \; ;$$

- The Fermi constant measured from the muon lifetime,

$$G_F(\mu) = 1.16639(1) \times 10^{-5} \text{ GeV}^{-2} \; ;$$

- The Z boson mass that was obtained by LEP at the Z pole,

$$M_Z = 91.1867(21) \text{ GeV} \; .$$

These input parameters can be written, at tree level, in terms of just g, g' and v as

$$\begin{aligned} \alpha_0(0) &= \frac{g^2 s_W^2}{4\pi} \; , \\ G_{F_0} &= \frac{1}{\sqrt{2} v^2} \; , \\ M_{Z_0}^2 &= \frac{g^2 v^2}{4 c_W^2} \; . \end{aligned} \tag{3.1}$$

where the subscript 0 indicates that these relations are valid at tree level, and

$$s_W^2 \equiv \frac{g'^2}{g^2 + g'^2} \quad , \quad \text{and } \; c_W^2 \equiv \frac{g^2}{g^2 + g'^2} \; ,$$

depend only on g and g':

3.1.1 One Loop Calculations

Let us write the vacuum polarization amplitude (self–energy) for vector bosons ($a, b = \gamma, W, Z$) as

$$\Pi_{ab}^{\mu\nu}(q^2) \equiv g^{\mu\nu} \, \Pi_{ab}(q^2) + (q^\mu q^\nu \text{ terms}) \; .$$

The terms proportional to $q^\mu q^\nu$ can be dropped since these amplitudes should be plugged in conserved fermion currents, and from the Dirac equation, they will give rise to terms that goes with the external fermion mass that can be neglected in the usual experimental situation.

We can summarize the relevant quantities for the loop corrections of the Standard Model [103]:

- The vector and axial form factors of the Z^0 coupling, at $q^2 = M_Z^2$, which include both the vertex and the fermion self–energy radiative corrections. From (2.21) we can write

  $$V^\mu_{Zf\bar{f}} = -i \frac{g}{2\cos\theta_W} \bar{\psi}_f \gamma^\mu (g_V^f - g_A^f \, \gamma_5) \psi_f \; ,$$

  where (2.23) and (2.24) now are given by

  $$\begin{aligned} g_V^f &= \sqrt{\rho} \, \left(T_3^f - 2 \, \kappa_f \, Q_f \sin^2\theta_W \right) \; , \\ g_A^f &= \sqrt{\rho} \, T_3^f \; . \end{aligned} \qquad (3.2)$$

  which define the relative strength of the neutral and charged currents, ρ, and the coefficient of the $\sin^2\theta_W$, κ_f. Notice that at tree level, $\rho = \kappa_f = 1$.

- Correction to μ–decay amplitude at $q^2 = 0$, which includes the box (B), vertex (V) and the fermion self–energy corrections

$$\mathcal{M}(\mu) = -i\,\delta G_{(B,V)} \left[\bar{\psi}_e \gamma^\mu (1-\gamma_5)\psi_{\nu_e}\right] \left[\bar{\psi_{\nu_\mu}} \gamma^\mu (1-\gamma_5)\psi_\mu\right] \ .$$

We can write the corrections to the input parameters as,

$$\begin{aligned} \alpha &= \alpha_0 + \delta\alpha \ , \\ M_Z^2 &= M_{Z_0}^2 + \delta M_Z^2 \ , \\ G_F &= G_{F_0} + \delta G_F \ , \end{aligned} \tag{3.3}$$

where, in terms of the vacuum polarization amplitude, and $\delta G_{(B,V)}$ the corrections become

$$\begin{aligned} \frac{\delta\alpha}{\alpha} &= -\Pi_{\gamma\gamma}(0) - 2\frac{s_W}{c_W}\frac{\Pi_{\gamma Z}(0)}{M_Z^2} \ , \\ \frac{\delta M_Z^2}{M_Z^2} &= -\frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} \ , \\ \frac{\delta G_F}{G_F} &= \frac{\Pi_{WW}(0)}{M_W^2} + \frac{\delta G_{(B,V)}}{G_F} \ . \end{aligned} \tag{3.4}$$

Correction to the Derived Observables

From the corrections to the input parameters we can estimate the radiative corrections to the derived observables. Let us write the tree level input variables α_0, G_{F_0}, and M_{Z_0} as $\mathcal{I}_0^i$. When we compute the radiative correction to the input parameters $\mathcal{I}_0^i$, we have

$$\mathcal{I}_0^i \ \longrightarrow \ \mathcal{I}^i(\mathcal{I}_0^i) = \mathcal{I}_0^i + \delta\mathcal{I}^i(\mathcal{I}_0^i) \ .$$

The relation for the renormalized input variables, $\mathcal{I}^i$, can be inverted to write $\mathcal{I}_0^i = \mathcal{I}_0^i(\mathcal{I}^i)$.

The same holds true for any derived observable ($\mathcal{O}$) or any S–matrix element, that is,

$$\begin{aligned} \mathcal{O}[\mathcal{I}_0^i(\mathcal{I}^i)] &= \mathcal{O}_0(\mathcal{I}_0^i) + \delta\mathcal{O}(\mathcal{I}_0^i) \\ &= \mathcal{O}_0(\mathcal{I}^i - \delta\mathcal{I}^i) + \delta\mathcal{O}(\mathcal{I}^i - \delta\mathcal{I}^i) \\ &= \mathcal{O}_0(\mathcal{I}^i) - \sum_i \frac{\partial\mathcal{O}_0}{\partial\mathcal{I}^i}\,\delta\mathcal{I}^i + \delta\mathcal{O}(\mathcal{I}^i) \\ &\equiv \mathcal{O}_0(\mathcal{I}^i) + \Delta\mathcal{O}(\mathcal{I}^i) \ . \end{aligned} \tag{3.5}$$

At one loop it is enough to renormalize just the input variables $\mathcal{I}^i$. However, at two loops it is necessary also to renormalize all other parameters that intervene at one loop level like m_t and M_H.

As an example, let us compute the radiative correction to the W boson mass. At tree level M_W is given by [see (2.31)]

$$M_{W_0}^2 = c_{W_0}^2 M_{Z_0}^2 \ .$$

Writing c_W^2 in terms of the input variables, we have

$$c_W^2 = (1 - s_W^2) = \left[1 - \left(\frac{4\pi\alpha}{g^2}\right)\right] = \left[1 - \left(\frac{\pi\alpha}{\sqrt{2} G_F M_Z^2 c_W^2}\right)\right] \ .$$

Solving for c_W^2, we get

$$M_{W_0}^2(\mathcal{I}^i) = \left\{\frac{1}{2}\left[1 + \left(1 - \frac{4\pi\alpha}{\sqrt{2} G_F M_Z^2}\right)^{1/2}\right]\right\} M_Z^2 \ .$$

Taking into account the derivatives,

$$\begin{aligned}
\frac{\partial M_{W_0}^2}{\partial \alpha} &= \frac{s_W^2 c_W^2}{s_W^2 - c_W^2} \frac{M_Z^2}{\alpha} \ , \\
\frac{\partial M_{W_0}^2}{\partial G_F} &= -\frac{s_W^2 c_W^2}{s_W^2 - c_W^2} \frac{M_Z^2}{G_F} \ , \\
\frac{\partial M_{W_0}^2}{\partial M_Z^2} &= -\frac{c_W^4}{s_W^2 - c_W^2} \ .
\end{aligned}$$

We obtain from (3.5) the M_W correction

$$\begin{aligned}
M_W^2 &= M_{W_0}^2(\mathcal{I}^i) - \sum_i \frac{\partial M_{W_0}^2}{\partial \mathcal{I}^i} \delta\mathcal{I}^i + \delta M_W^2(\mathcal{I}^i) \\
&= c_W^2 M_Z^2 - \frac{c_W^2 M_Z^2}{s_W^2 - c_W^2}\left(s_W^2 \frac{\delta\alpha}{\alpha} - s_W^2 \frac{\delta G_F}{G_F} - c_W^2 \frac{\delta M_Z^2}{M_Z^2}\right) + \delta M_W^2 \ ,
\end{aligned}$$

with

$$\delta M_W^2 = -\Pi_{WW}(M_W^2) \ .$$

3.2 The Z boson Physics

3.2.1 Introduction

The most important experimental tests of the Standard Model in this decade was performed at the Z pole. The four LEP Collaborations (Aleph, Delphi, L3, and Opal)[105] and the SLAC SLD Collaboration [106] studied the reaction,

$$e^+e^- \to Z^0 \to f\bar{f} \; .$$

The main purpose of these experiments was to test the Standard Model at the level of its quantum corrections and also to try to obtain some hint on the top quark mass and on the Higgs boson.

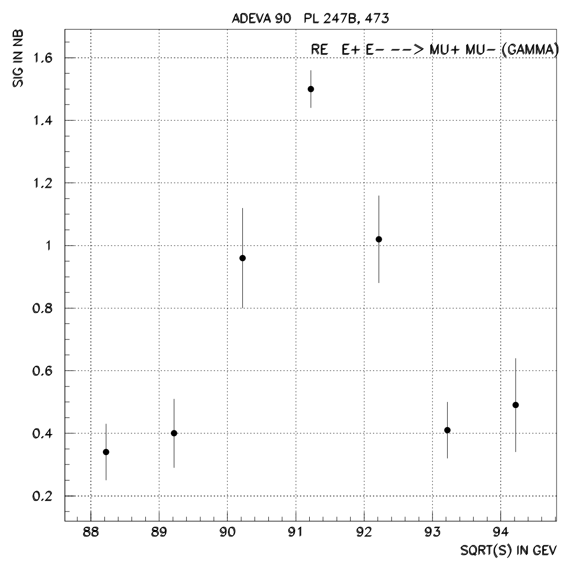


Fig. 6: The Z profile measured by the L3 Collab. [107]

At CERN, after scanning the Z resonance (see Fig. 6), data were collected at the Z peak, and around 17 millions of Z's were produced and studied.

The shape of the resonance is characterised by the cross section for the fermion pair ($f\bar{f}$) production at the Z peak,

$$\sigma^0_{f\bar{f}} = \frac{12\pi}{M_Z^2}\frac{\Gamma_e\,\Gamma_f}{\Gamma_Z^2}\ ,$$

where the position of the peak gives the value $M_Z = 91186.7 \pm 2.1$ MeV, the full width at half maximum (FWHM) represents the Z width, $\Gamma_Z = 2493.9 \pm 2.4$ MeV, and the height of the peak gives the value of the total cross section for $f\bar{f}$ production, $\sigma^0_{f\bar{f}}$.

For the analysis of the Z physics it is necessary to choose the input parameters at the appropriate scale, M_Z. The relative uncertainty of the input parameters are:

Parameter	Value	Uncertainty
m_t [108]	174.3 ± 5.1 GeV	2.9 %
$\alpha_s(M_Z^2)$ [105]	0.119 ± 0.002	1.7 %
$\alpha^{-1}(M_Z^2)$ [109, 110]	128.878 ± 0.090	0.07 %
M_Z [105]	91186.7 ± 2.1 MeV	0.0023 %
$G_F(\mu)$ [32]	$(1.16639 \pm 0.00001) \times 10^{-5}$ GeV^{-2}	0.00086 %

Table I: Relative uncertainty of the input parameters.

For the Higgs boson mass we just have available a lower bound of $M_H > 95.2$ GeV at 95% of C.L. [111].

Notice that, in spite of the very precise measurement of the electromagnetic structure constant at low energy, which has a relative uncertainty of just 0.0000045 %, its value at M_Z is much less precise. This uncertainty arises from the contribution of light quarks to the vacuum polarization. The evolution of α is given by

$$\alpha(M_Z^2) = \frac{\alpha(0)}{1-\Delta\alpha}\ , \tag{3.6}$$

where

$$\Delta\alpha = \Delta\alpha_{\rm lep} + \Delta\alpha_{\rm had} + \Delta\alpha_{\rm top} \; .$$

The top quark contribution is proportional to $1/m_t^2 \sim 10^{-5}$. The contributions from leptons (ℓ) [110] and light quarks (q) [109] are:

$$\begin{aligned} \Delta\alpha_{\rm lep} &= 0.031498 \; , \\ \Delta\alpha_{\rm had} &= -(0.02804 \pm 0.00065) \; . \end{aligned} \tag{3.7}$$

where the error in $\Delta\alpha_{\rm lep}$ is negligible. Therefore, the loss of precision comes from $\Delta\alpha_{\rm had}$ due to non–perturbative QCD effects that are large at low energies and to the imprecision in the light quark masses.

Other important pure QED corrections are the initial and final state photon radiation. The initial state radiation is taken into account by convoluting the cross section with the radiator function $H(k)$,

$$\sigma(s) = \int_0^{k_{\rm max}} dk \; H(k)\sigma_0[s(1-k)] \; ,$$

where $k_{\rm max}$ represents a cut in the maximum radiated energy. The radiator function takes into account virtual and real photon emissions and includes soft photon resummation [112].

The final state radiation is included by multiplying the bare cross sections and widths by the QED correction factor,

$$\left(1 + \frac{3\alpha Q_f^2}{4\pi}\right) \simeq (1 + 0.002 \; Q_f^2) \; .$$

where Q_f is the fermion charge.

3.2.2 The Standard Model Parameters

We present in the following sections the Standard Model predictions for some observables. We compare these predictions with the values measured by the CERN LEP and at the SLAC SLD Collaborations, and stress the very impressive agreement between them.

Z Partial Widths

The Z width into a fermion pair, at tree level, is given in the Standard Model by,

$$\Gamma(Z \to f\bar{f}) = C \frac{G_F M_Z^3}{6\sqrt{2}\pi} \left[(g_A^f)^2 + (g_V^f)^2 \right] \ , \tag{3.8}$$

where C refers to the fermion color, *i.e.*,

$$C = \begin{cases} 1 \ , & \text{for leptons} \ , \\ 3\left[1 + \alpha_s(M_Z)/\pi + 1.409\alpha_s^2(M_Z)/\pi^2 + \cdots\right] \ , & \text{for quarks} \ . \end{cases}$$

where the QCD corrections were included for quarks. At loop level we should consider the modifications to g_V^f and g_A^f (3.2) and the appropriate QED corrections discussed in the last section.

The value of the partial width for the different fermion flavors are

$f\bar{f}$ Pair	Partial Width
$\nu\bar{\nu}$	167.25 MeV
e^+e^-	84.01 MeV
$u\bar{u}$	300.30 MeV
$d\bar{d}$	383.10 MeV
$b\bar{b}$	376.00 MeV

Table II: $Z \to f\bar{f}$ partial widths.

The experimental results for the partial widths are [105],

$$\begin{aligned} \Gamma_\ell &\equiv \Gamma(Z \to \ell^+\ell^-) = 83.90 \pm 0.10 \text{ MeV} \ , \\ \Gamma_{\text{had}} &\equiv \Gamma(Z \to \text{hadrons}) = 1742.3 \pm 2.3 \text{ MeV} \ , \\ \Gamma_Z &\equiv \Gamma(Z \to \text{all}) = 2493.9 \pm 2.4 \text{ MeV} \ , \\ \Gamma_{\text{inv}} &\equiv \Gamma_Z - 3\,\Gamma_\ell - \Gamma_{\text{had}} = 500.1 \pm 1.9 \text{ GeV} \ , \end{aligned}$$

where we assume three leptonic channels (e^+e^-, $\mu^+\mu^-$, and $\tau^+\tau^-$), and Γ_{inv} is the invisible Z width.

Number of Neutrino Species

We can extract information on the number of light neutrino species by supposing that they are responsible for the invisible width, *i.e.* $\Gamma_{\text{inv}} = N_\nu \Gamma_\nu$. The LEP data [105] gives the ratio of the invisible and leptonic Z partial widths, $\Gamma_{\text{inv}}/\Gamma_\ell = 5.961 \pm 0.023$. On the other hand, Standard Model predicts the $(\Gamma_\nu/\Gamma_\ell)_{\text{SM}} = 1.991 \pm 0.001$, where the error is associated to the variation of m_t and M_H. In the ratio of these two expressions, Γ_ℓ cancels out and yields the number of neutrino species,

$$N_\nu = 2.994 \pm 0.011 \ ,$$

where N_ν represents the total number of neutrino flavors that are accessible kinematically to the Z, that is $M_\nu < M_Z/2$. This result indicates that, if the observed pattern of the first three generations is assumed, then we have only these families of fermions in nature.

Radiative Corrections Beyond QED

An important question to be asked when comparing the Standard Model predictions with experimental data is if the effect of pure weak radiative correction could indeed be measured. This question can be answered by looking, for instance, at the plot of $\sin^2\theta_{\text{eff}} \times \Gamma_\ell$ (Fig. 7), where Γ_ℓ is given by (3.8) and,

$$\sin^2\theta_{\text{eff}} \equiv \frac{1}{4}\left(1 - \frac{g_V^\ell}{g_A^\ell}\right) = 0.23157 \pm 0.00018 \ .$$

The point at the lower–left corner shows the prediction when only the QED (photon vacuum polarization) correction is included and the respective variation for $\alpha(M_Z^2)$ varying by one standard deviation. The Standard Model prediction, with the full (QED + weak) radiative correction, is represented by the band that reflects the dependence on the Higgs (95 GeV $< M_H <$ 1000 GeV) and on the top mass (169.2 GeV $< m_t <$ 179.4 GeV). We notice that the presence of genuine electroweak correction is quite evident.

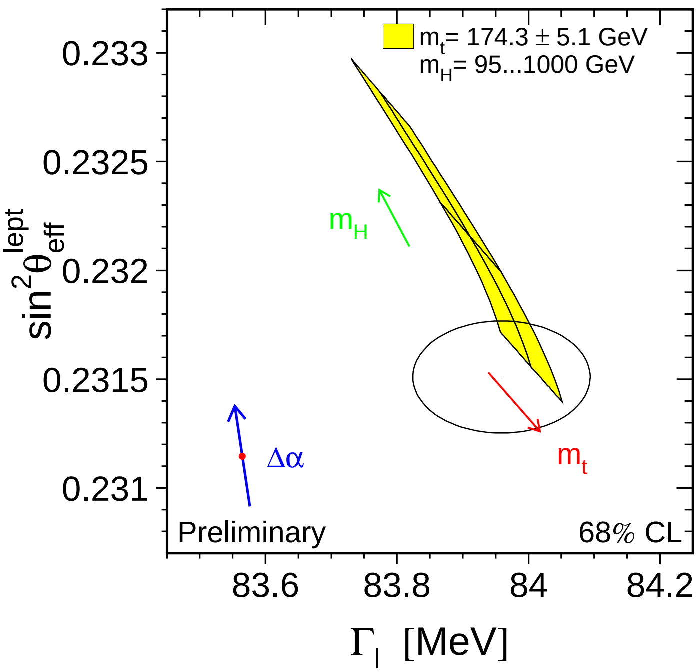


Fig. 7: LEP + SLD measurements of $\sin^2\theta_{\rm eff}$ *and* Γ_ℓ*, compared to the Standard Model prediction [113].*

Another important evidence for pure electroweak correction comes from the radiative correction Δr_W to the relation between M_W and G_F,

$$\left(1-\frac{M_W^2}{M_Z^2}\right)\frac{M_W^2}{M_Z^2}=\frac{\pi\alpha(M_Z^2)}{\sqrt{2}G_F M_Z^2(1-\Delta r_W)}\ , \tag{3.9}$$

where $\alpha(M_Z^2)$ is given by (3.6), and therefore, the effect of the running of α was subtracted in the definition of Δr_W. Taking into account the value measured at LEP and Tevatron, $M_W = 80.394 \pm 0.042$ GeV, we have [104]: $(\Delta r)_W^{\rm exp} = -0.02507 \pm 0.00259$. Thus, the correction representing only the electroweak contribution, not associated with the running of α, is $\sim 10\,\sigma$ different from zero.

g_V^ℓ, g_A^ℓ, and the Lepton Universality

The partial Z width in the different lepton flavors is able to provide a very important information on the universality of the electroweak interactions. The values of g_V^ℓ and g_A^ℓ can be plotted for $\ell = e$, μ and τ.

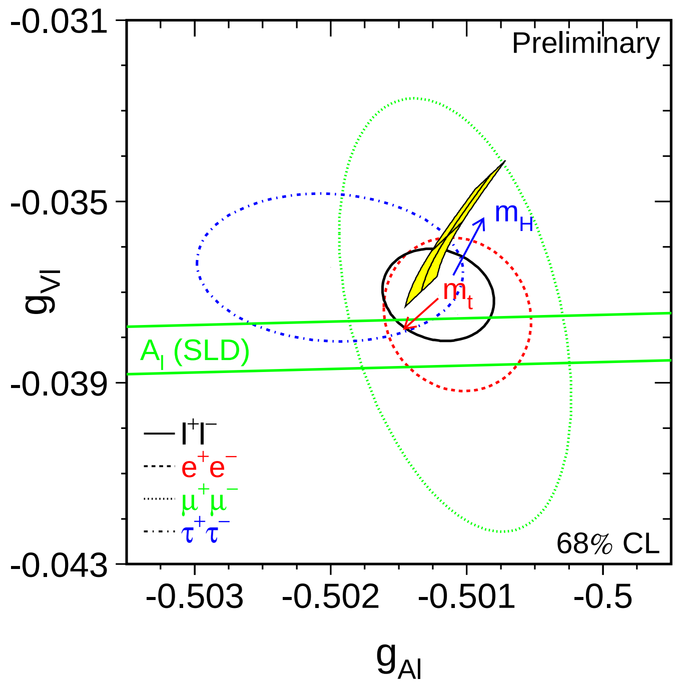


Fig. 8: 68% C.L. contours in the $g_V^\ell \times g_A^\ell$ plane. The solid line is a fit assuming lepton universality. The band corresponds to the SLD result from A_{LR} (3.12) measurements [113].

The result present in Fig. 8 shows that the measurements of $g_V^\ell \times g_A^\ell$ are consistent with the hypothesis that the electroweak interaction is universal and yields

$$g_V^\ell = -0.03703 \pm 0.00068 \ , \quad g_A^\ell = -0.50105 \pm 0.00030 \ .$$

Notice that the value of g_A^ℓ disagrees with the Born prediction of -0.5 (2.24) by 3.5 σ. However they are in very good agreement with the Standard Model values [32]: $(g_V^\ell)^{\rm SM} = -0.0397 \pm 0.0003$ and $(g_A^\ell)^{\rm SM} = -0.5064 \pm 0.0001$. This is another important evidence of the weak radiative corrections.

Asymmetries

Since parity violation comes from the difference between the right and left couplings of the Z^0 to fermions, it is convenient to define the combination of the vector and axial couplings of the fermions as

$$A_f = \frac{2 g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2} \; . \tag{3.10}$$

The events $e^+e^- \to f^+f^-$ can be characterized by the momentum direction of the emitted fermion. We say that the final state fermion (f^-) travels forward (F) or backward (B) with respect to the electron (e^-) beam. Therefore, we can define the forward–backward asymmetry by

$$A_{FB} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \; ,$$

and at the Z pole, this asymmetry is given by

$$A_{FB}^{0,f} = \frac{3}{4} A_e A_f \; . \tag{3.11}$$

The measurement of $A_{FB}^{0,f}$ for charged leptons, and c and b quarks give us information only on the product of A_e and A_f. On the other hand, the measurement of the τ lepton polarization is able to determine the values of A_e and A_τ separately. The longitudinal τ polarization is defined as

$$\mathcal{P}_\tau \equiv \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \; ,$$

where $\sigma_{R(L)}$ is the cross section for tau–lepton pair production of a right (left) handed τ^-. At the Z pole, $\mathcal{P}_\tau$ can be written in terms of scattering (e^-, τ^-) angle θ as,

$$\mathcal{P}_\tau = -\frac{A_\tau(1+\cos^2\theta) + 2A_e\cos\theta}{1+\cos^2\theta + 2A_eA_\tau\cos\theta} \ .$$

This yields [105]

$$A_e = 0.1479 \pm 0.0051 \quad , \quad A_\tau = 0.1431 \pm 0.0045 \ ,$$

which are in agreement with the lepton universality ($A_\ell = 0.1469 \pm 0.0027$). They are also in agreement with the Standard Model prediction: $A_\ell^{\rm SM} = 0.1475 \pm 0.0013$.

This result can be used to extract information on the heavy quark couplings: $A_c = 0.646 \pm 0.043$ and $A_b = 0.899 \pm 0.025$, which should be compared with the Standard Model values of $A_c^{\rm SM} = 0.6679 \pm 0.0006$ and $A_b^{\rm SM} = 0.9348 \pm 0.0001$.

Another interesting asymmetry that can be measured by SLD is the left–right cross section asymmetry,

$$A_{\rm LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \ , \qquad (3.12)$$

where $\sigma_{L(R)}$ is the cross section for (left–) right–handed incident electron with the positron kept unpolarized. Since, at the Z pole, $A_{\rm LR} = A_e$, we can get the best measurement of the electron couplings: $A_e = 0.1510 \pm 0.0025$ (see Fig. 8).

Higgs Mass Sensitivity

In order to give an idea of the sensitivity of the different electroweak observables to the Higgs boson mass, we compare in Fig. 9 the experimental values with the the Standard Model theoretical predictions, as a function of M_H.

The vertical band represents the experimental measurement with the respective error. The theoretical prediction includes the errors in $\alpha(M_Z^2)$, from $\Delta\alpha_{\rm had}$ (3.7), $\alpha_s(M_Z^2)$, and m_t (see Table I).

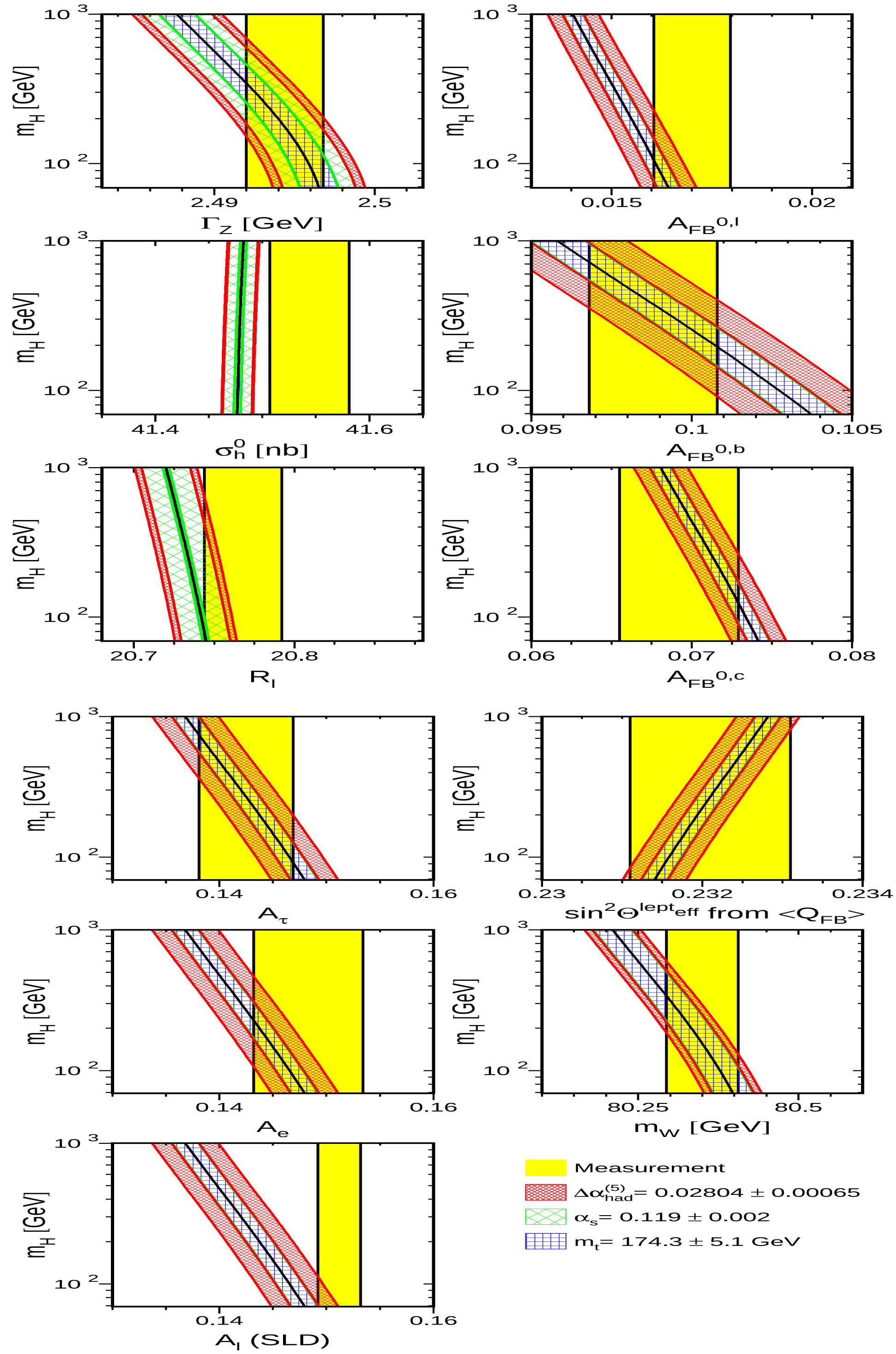


Fig. 9: LEP measurements compared with the Standard Model predictions, as a function of M_H [113].

In this figure σ_h^0 is the hadronic cross section at the Z pole, $R_\ell \equiv (\Gamma_{\rm had}/\Gamma_\ell)$. $A_{FB}^{0,f}$ is defined in (3.11) and A_f in (3.10). $< Q_{FB} >$ is the average charge, which is related to the forward–backward asymmetries by

$$< Q_{FB} >= \sum_q \delta_q \, A_{FB}^q \, \frac{\Gamma_{q\bar{q}}}{\Gamma_{\rm hadr}} \, ,$$

where δ_q is the average charge difference between the q and $\bar{q}$ hemispheres. For the sake of comparison A_e, extracted by SLD from A_{LR} (3.12), is also shown. We can see from Fig. 9 that dependence on the Higgs mass varying in the range 95 GeV $< M_H <$ 1000 GeV is quite mild for all the observables, since the Higgs effect enters only via $\log(M_H^2/M_Z^2)$ factors.

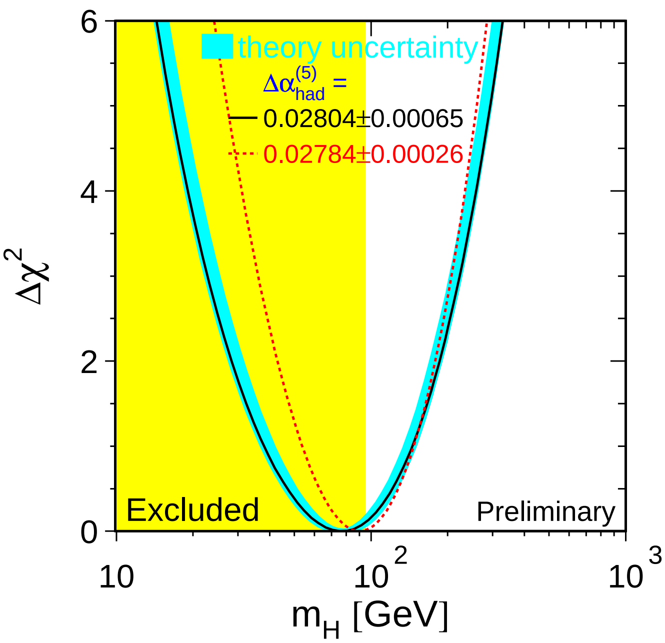


Fig. 10: $\Delta\chi^2 \equiv \chi^2 - \chi^2_{\rm min}$ *as a function of* M_H *[105, 113].*

However we can extract information on M_H from the global fit including all data on the different observables. In Fig. 10 we show the plot of $\Delta\chi^2 \equiv \chi^2 - \chi^2_{\min}$ versus M_H. The left vertical band represents the excluded region due to the direct search for the Higgs ($M_H \gtrsim 95$ GeV). The band represents an estimate of the theoretical error due to missing higher order corrections. The global fit results in $M_H = 91^{+64}_{-41}$ GeV.

	Measurement	Pull
m_Z [GeV]	91.1871 ± 0.0021	.08
Γ_Z [GeV]	2.4944 ± 0.0024	-.56
σ^0_{hadr} [nb]	41.544 ± 0.037	1.75
R_e	20.768 ± 0.024	1.16
$A^{0,e}_{fb}$	0.01701 ± 0.00095	.80
A_e	0.1483 ± 0.0051	.21
A_τ	0.1425 ± 0.0044	-1.07
$\sin^2\theta^{lept}_{eff}$	0.2321 ± 0.0010	.60
m_W [GeV]	80.350 ± 0.056	-.62
R_b	0.21642 ± 0.00073	.81
R_c	0.1674 ± 0.0038	-1.27
$A^{0,b}_{fb}$	0.0988 ± 0.0020	-2.20
$A^{0,c}_{fb}$	0.0692 ± 0.0037	-1.23
A_b	0.911 ± 0.025	-.95
A_c	0.630 ± 0.026	-1.46
$\sin^2\theta^{lept}_{eff}$	0.23099 ± 0.00026	-1.95
$\sin^2\theta_W$	0.2255 ± 0.0021	1.13
m_W [GeV]	80.448 ± 0.062	1.02
m_t [GeV]	174.3 ± 5.1	.22
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02804 ± 0.00065	-.05

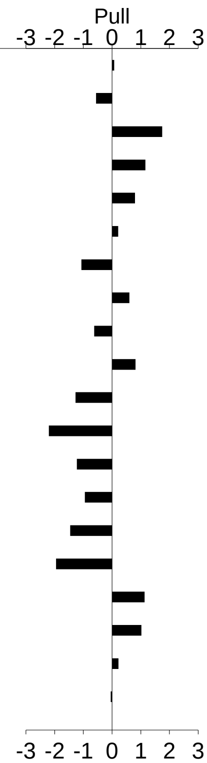


Fig. 11: Comparison of the precision electroweak measurements with the Standard Model predictions [113].

As a final comparison, we present in Fig. 11 a list of several electroweak observables. The experimental values are compared with the Standard Model theoretical predictions. The Pull $\equiv (O_{\rm meas} - O_{\rm fit})/\sigma_{\rm meas}$, represents the number of standard deviations that separate the central values. This results show an impressive agreement with the Standard Model expectations.

Chapter 4

The Higgs Boson Physics

4.1 Introduction

The procedure of generating vector boson masses in a gauge invariant way requires the introduction of a complex doublet of scalar fields (2.25) which corresponds to four degrees of freedom. Three out of these are "eaten" by the gauge bosons, W^+, W^-, and Z^0, and become their longitudinal degree of freedom. Therefore, it remains in the physical spectrum of the theory the combination

$$\frac{(\phi^0 + \bar{\phi}^0)}{\sqrt{2}} = H + v ,$$

where v is given by (2.27), and H is the physical Higgs boson field.

The Higgs boson mass (2.34) can be written as

$$M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda}\, v = \left(\frac{\sqrt{2}}{G_F}\right)^{1/2} \sqrt{\lambda}\, . \tag{4.1}$$

Both Higgs potential parameters, μ^2 and λ, are *a priori* unknown — just their ratio is fixed by the low energy data [see (2.17) and (2.31)]. Therefore the Standard Model does not provide any direct information on the Higgs boson mass.

The discovery of this particle is one of the challenges of the high–energy colliders. This is the most important missing piece of the Standard Model and its experimental verification could furnish very important information on the spontaneous breaking of the electroweak symmetry and on the mechanism for generating fermion masses. The phenomenology of the Standard Model Higgs boson is covered in great detail in reference [114]. Recent review articles include Ref. [115], [116], [117], [118]. We intend to emphasize here the most relevant properties of Higgs particle and make a brief summary of the prospects for its search in the near future.

4.2 Higgs Boson Properties

The Higgs Couplings

The Higgs boson couples to all particles that get mass ($\propto v$) through the spontaneous symmetry breaking of $SU(2)_L \otimes U(1)_Y$. We collect in Table III the intensity of the coupling to the different particles from (2.30), (2.33), (2.36), and (2.46),

Coupling	Intensity
$Hf\bar{f}$	M_f/v
HW^+W^-	$2M_W^2/v$
HZ^0Z^0	M_Z^2/v
HHW^+W^-	M_W^2/v^2
HHZ^0Z^0	$M_Z^2/2v^2$
HHH	$M_H^2/2v$
$HHHH$	$M_H^2/8v^2$

Table III: The Higgs coupling to different particles.

From the results of Table III it becomes evident that the Higgs couples proportionally to the particle masses. Therefore we can establish two general principles that should guide the search of the Higgs boson:

(i) it will be produced in association with heavy particles; (ii) it will decay into the heaviest particles that are accessible kinematically.

Besides the couplings presented in Table III, the Higgs can also couple to $\gamma\gamma$ [119], $Z\gamma$ [120, 121] and also to gluons [122, 123], at one loop level. The neutral and weak interacting Higgs boson can interact with photons through loops of charged particles that share the weak and electromagnetic interactions: leptons, quarks and W boson. In the same way the Higgs couples indirectly with the gluons via loops of (weak and strong interacting) quarks.

Bounds on the Higgs Boson Mass

Since the Higgs boson mass is not predicted by the model we should rely on some experimental and theoretical bounds to guide our future searches. The most stringent lower bound was recently established by the LEP Collaborations [111] and read

$$M_H > 95.2 \text{ GeV} .$$

at 95% C.L..

It is also possible to obtain a theoretical lower bound on the Higgs boson mass based on the stability of the Higgs potential when quantum corrections to the classical potential (2.26) are taken into account [124]. Requiring that the standard electroweak minimum is stable (*i.e.* the vacuum is an absolute minimum) up to the Planck scale, $\Lambda = 10^{19}$ GeV, the following bound can be established [125]:

$$M_H \text{ (in GeV)} > 133 + 1.92(m_t - 175) - 4.28 \left(\frac{\alpha_s - 0.12}{0.006} \right) .$$

The behavior of M_H as a function of the scale Λ is given in the lower curve of Fig. 13, for $m_t = 175$ GeV and $\alpha_s = 0.118$. We see from this figure that, if a Higgs boson is discovered with $M_H \simeq 100$ GeV, it would mean that the electroweak vacuum is instable at $\Lambda \sim 10^5$ GeV *.

*Reversing the argument, since we live in a stable vacuum, this means that the Standard Model must break down at this same scale.

There are also some theoretical upper bounds on the Higgs boson mass. A bound can be obtained by requiring that unitarity is not violated in the scattering of vector bosons [126]. Let us take as an example the WW scattering represented in Fig. 12.

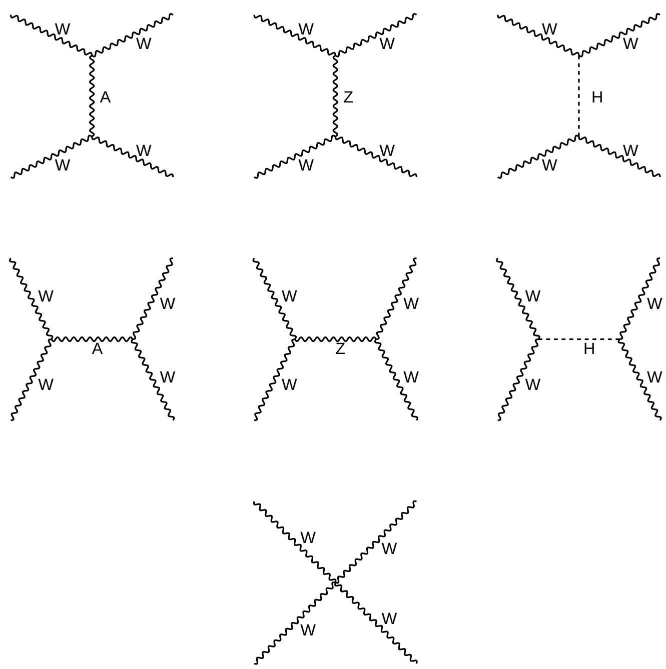


Fig. 12: Feynman contributions to $W^+W^- \to W^+W^-$.

We should notice that if we exclude the Higgs boson contribution by taking $M_H \to \infty$, we expect that the remaining amplitudes would eventually violate unitarity, since the theory is not renormalisable without the Higgs. Therefore, it is natural to expect that the Higgs mass should play an important rôle in high energy behaviour of the scattering amplitudes of longitudinally polarized vector bosons. This is what happened for instance in the reaction $e^+e^- \to W^+W^-$ discussed in section 2.2.

A convenient way to estimate amplitudes involving longitudinal gauge bosons is through the use of the Goldstone Boson Equivalence The-

orem [126, 127]. This theorem states that at high energies, the amplitude $\mathcal{M}$ for emission or absorption of a longitudinally polarized gauge boson is equal to the amplitude for emission or absorption of the corresponding Goldstone boson, up to terms that fall like $1/E^2$, *i.e.*,

$$\mathcal{M}(W_L^{\pm}, Z_L^0) = \mathcal{M}(\omega^{\pm}, z^0) + \mathcal{O}\left(M_{W,Z}^2/E^2\right) \ . \tag{4.2}$$

We can use an effective Lagrangian approach to describe the Goldstone boson interactions. Starting from the Higgs doublet in terms of $\omega^{\pm}$ and z^0,

$$\Phi = \frac{1}{\sqrt{2}} \left(\begin{array}{c} i\sqrt{2}\omega^+ \\ v + H - iz^0 \end{array} \right) \ ,$$

we can write the Higgs potential as

$$\begin{aligned} V(\Phi^\dagger\Phi) &= \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 \\ &= \frac{1}{2}M_H^2 H^2 + \frac{g}{4}\frac{M_H^2}{M_W} H(H^2 + 2\omega^+\omega^- + z^{0^2}) \\ &\quad + \frac{g^2}{32}\frac{M_H^2}{M_W^2}(H^2 + 2\omega^+\omega^- + z^{0^2})^2 \ . \end{aligned}$$

Therefore, with the aid of (4.2), the amplitude for $W_L^+W_L^- \to W_L^+W_L^-$ is obtained as,

$$\begin{aligned} \mathcal{M}(W_L^+W_L^- \to W_L^+W_L^-) &\simeq \mathcal{M}(\omega^+\omega^- \to \omega^+\omega^-) \\ &= -i\,\frac{g^2}{4}\frac{M_H^2}{M_W^2}\left(2 + \frac{M_H^2}{s - M_H^2} + \frac{M_H^2}{t - M_H^2}\right) \ . \end{aligned}$$

and, at high energies, we have:

$$\mathcal{M}(\omega^+\omega^- \to \omega^+\omega^-) \simeq -i\,\frac{g^2}{2}\frac{M_H^2}{M_W^2} = -i\;2\sqrt{2}G_F M_H^2 \ .$$

Therefore, for s-wave, unitarity requires

$$A_0 = \frac{1}{16\pi}|\mathcal{M}(\omega^+\omega^- \to \omega^+\omega^-)| = \frac{2G_F}{8\pi\sqrt{2}}M_H^2 < 1 \ .$$

When this result is combined with the other possible channels ($z^0 z^0$, $z^0 h$, hh) it leads to the requirement that $\lambda \lesssim 8\pi/3$ or, translated in terms of the Higgs mass,

$$M_H \lesssim \left(\frac{8\pi\sqrt{2}}{3G_F}\right)^{1/2} \simeq 1\,\text{TeV} .$$

Another way of imposing a bound on the Higgs mass is provided by the analysis of the triviality of the Higgs potential [124]. The renormalization group equation, at one loop, for the quartic coupling λ is

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\left(12\lambda^2\right) + \;(\text{terms involving } g, g', \text{Yukawa}) ,$$

where $t = \log(Q^2/\mu^2)$. Therefore, for a pure ϕ^4 potential, *i.e.*, when the gauge and Yukawa couplings are neglected, we have the solution

$$\frac{1}{\lambda(\mu)} - \frac{1}{\lambda(Q)} = \frac{3}{4\pi^2}\log\left(\frac{Q^2}{\mu^2}\right) .$$

Since the stability of the Higgs potential requires that $\lambda(Q) \geq 0$, we can write

$$\lambda(\mu) \leq \frac{4\pi^2}{3\log(Q^2/\mu^2)} , \tag{4.3}$$

and, for large values of Q^2, we can see that $\lambda(\mu) \to 0$ and the theory becomes trivial, that is, not interacting. The relation (4.3), can be written as

$$Q^2 \leq \mu^2 \exp\left[\frac{4\pi^2}{3\lambda(\mu)}\right] .$$

This result gives rise to a bound in the Higgs boson mass when we consider the scale $\mu^2 = M_H^2$ and take into account (4.1),

$$Q^2 \leq M_H^2 \exp\left[\frac{8\pi^2 v^2}{3M_H^2}\right] .$$

Therefore, there is a maximum scale $Q^2 = \Lambda^2$, for a given Higgs boson mass, up to where the Standard Model theory should be valid.

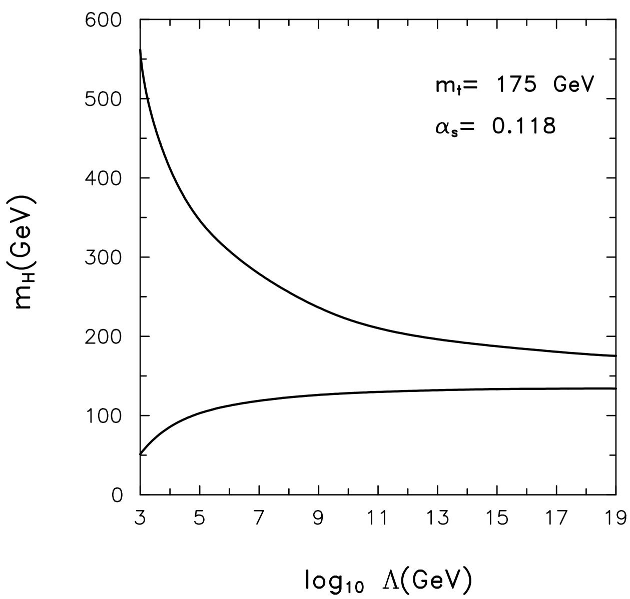


Fig. 13: Perturbative and stability bound on M_H as a function of the scale Λ, from Ref. [125].

In Fig. 13, we present the stability bound (lower curve) and the triviality bound (upper curve) on the Higgs boson mass as a function of the scale Λ. If we expect that the Standard Model is valid up to a given scale — let us say $\Lambda_{\rm GUT} \sim 10^{16}$ GeV [128] — a bound on the Higgs mass should lie between both curves, in this case 140 GeV $\lesssim M_H \lesssim 180$ GeV.

4.3 Production and Decay Modes

4.3.1 The Decay Modes of the Higgs Boson

The possible decay modes of the Higgs boson are essentially determined by the value of its mass. In Fig. 14 we present the Higgs branching ratio for different M_H.

When the Higgs boson mass lies in the range 95 GeV $< M_H < 130$ GeV, the Higgs is quite narrow $\Gamma_H < 10$ MeV and the main branching

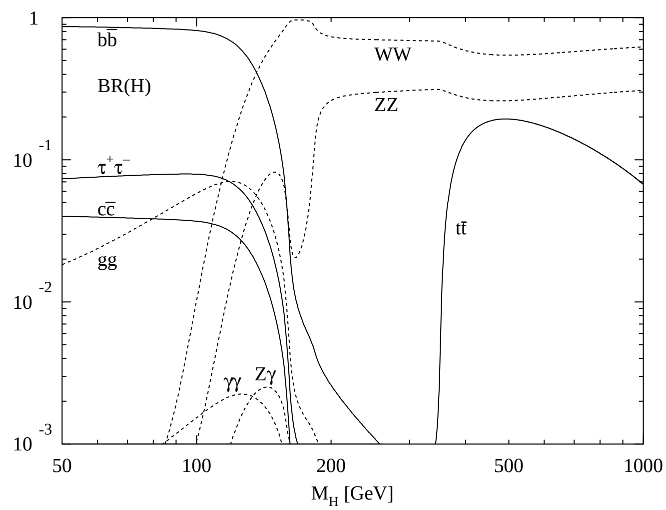


Fig. 14: The branching ratios of the Higgs boson as a function of its mass from Ref. [117].

ratios come from the heaviest fermions that are accessible kinematically:

$$\begin{aligned} BR(H \to b\bar{b}) &\sim 90\% \,, \\ BR(H \to c\bar{c}) &\simeq BR(H \to \tau^+\tau^-) \sim 5\% \,. \end{aligned}$$

For $M_H \simeq 120$ GeV the gluon–gluon channel is important giving a contribution of $\sim 5\%$ of the width. For a heavier Higgs, *i.e.* $M_H > 130$ GeV, the vector boson channels $H \to VV^*$, with $V = W$, Z, are dominant,

$$\begin{aligned} BR(H \to W^+W^-) &\sim 65\% \,, \\ BR(H \to Z^0Z^0) &\sim 35\% \,. \end{aligned}$$

For $M_H \simeq 500$ GeV the top quark pair production contributes with $\sim 20\%$ of the width. Note that the BR($H \to \gamma\gamma$) is always small $\mathcal{O}(10^{-3})$. However, we can think of some alternative models that give rise to larger $H\gamma\gamma$ couplings (for a review see [129] and references therein). For large values of M_H the Higgs becomes a very wide resonance: $\Gamma_H \sim [M_H$ (in TeV)$]^3/2$.

4.3.2 Production Mechanisms at Colliders

Electron–Positron Colliders

The Higgs boson can be produced in electron–positron collisions via the Bjorken mechanism [130] or vector boson fusion [131],

$$
\begin{aligned}
&(i)\ \text{Bjorken:} && e^+ + e^- \to Z \to Z\ H\ ,\\
&(ii)\ \text{WW Fusion:} && e^+ + e^- \to \nu\bar{\nu}(WW) \to \nu\bar{\nu}\ H\ ,\\
&(iii)\ \text{ZZ Fusion:} && e^+ + e^- \to e^+e^-(ZZ) \to e^+e^-\ H\ .
\end{aligned}
$$

At LEPI and II, where $\sqrt{s} \simeq M_Z$ or $2\,M_W$ the Higgs production is dominated by the Bjorken mechanism and they were able to rule out from very small Higgs masses up to 95.2 GeV [111]. Maybe, when the whole analysis is complete, they will be able to rule out up to $M_H \sim 106$ GeV.

At the future e^+e^- accelerators, like the Next Linear Collider [132], where $\sqrt{s} = 500$ GeV, the production of a Higgs with $100 < M_H < 200$ GeV will be dominated by the WW fusion, since its cross section behaves like $\sigma \propto \log(s/M_H^2)$ and therefore dominates at high energies. We can expect around 2000 events per year for an integrated luminosity of $\mathcal{L} \simeq 50$ fb^{-1}, and the Next Linear Collider should be able to explore up to $M_H \sim 350$ GeV.

Hadron Colliders

At proton–(anti)proton collisions the Higgs boson can be produced via the gluon fusion mechanism [122, 123], through the vector boson fusion and in association with a $W^\pm$ or a Z^0,

$$
\begin{aligned}
&(i)\ \text{Gluon fusion:} && p\bar{p} \to gg \to H\ ,\\
&(ii)\ \text{VV Fusion:} && p\bar{p} \to VV \to H\ ,\\
&(iii)\ \text{Association with V:} && p\bar{p} \to q\bar{q}' \to V\ H\ .
\end{aligned}
$$

At the Fermilab Tevatron [133], with $\sqrt{s} = 1.8$ (2) TeV, the Higgs is better produced in association with vector boson and they look for the $VH(\to b\bar{b})$ signature. After the improvement in the luminosity at TEV33 they will need $\mathcal{L} \sim 10$ fb^{-1} to explore up to $M_H \sim 100$ GeV with $5\,\sigma$.

At the CERN Large Hadron Collider [134], that will operate with $\sqrt{s} = 14$ TeV, the dominant production mechanism is through the gluon fusion and the best signature will be $H \to ZZ \to 4\,\ell^{\pm}$ for $M_H > 130$ GeV. For $M_H < 130$ GeV they should rely on the small BR$(H \to \gamma\gamma) \sim 10^{-3}$. We expect that the LHC can explore up to $M_H \sim 700$ GeV with an integrated luminosity of $\mathcal{L} \sim 100$ fb^{-1}.

Once the Higgs boson is discovered it is important to establish with precision several of its properties like mass, spin, parity and width. The next step would be to search for processes involving multiple Higgs production, like $VV \to HH$ or $gg \to HH$, which could give some information on the Higgs self–coupling.

Chapter 5

Closing Remarks

In the last 30 years, we have witnessed the striking success of a gauge theory for the electroweak interactions. The Standard Model made some new and crucial predictions. The existence of a weak neutral current and of intermediate vector bosons, with definite relation between their masses, were confirmed by the experiments.

Recently, a set of very precise tests were performed by Tevatron, LEP and SLC colliders that were able to reach an accuracy of 0.1% or even better, in the measurement of the electroweak parameters. This guarantees that even the quantum structure of the model was successfully confronted with the experimental data. It was verified that the W and Z couplings to leptons and quarks have exactly the same values anticipated by the Standard Model. We already have some strong hints that the triple–gauge–boson couplings respect the structure prescribed by the $SU(2)_L \otimes U(1)_Y$ gauge symmetry. The Higgs boson, remnant of the spontaneous symmetry breaking, has not yet been discovered. However, important information, extracted from the global fitting of data taking into account the loop effects of the Higgs, assures that this particle is just around the corner. The Higgs mass should be less than ~ 260 GeV at 95% C.L., in full agreement with the theoretical upper bounds for the Higgs mass.

These remarkable achievements let just a small room for the new physics beyond the Standard Model. Nevertheless, we still have some

conceptual difficulties like the hierarchy problem, that may indicate that the explanation provided by the Standard Model should not be the end of the story.

A series of alternative theories — technicolour, grand unified theory, supersymmetric extensions, superstrings, extra dimension theories, etc — have been proposed, but they all suffer from lack of an experimental spark. Nevertheless, the physics beyond the Standard Model is also beyond the scope of these lectures . . .

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